Emmy Noether

"Of all the women mathematicians, Emmy Noether is generally the best known. Often described as a loving, intelligent woman, she was impressive by many standards. She was faced with gender issues and political tensions in her lifetime, but her passion for mathematics remained strong."(Mishna par. 1)

Emmy Noether's early years saw her start down the path of learning. Emmy Noether was born on March 23, 1882 in Erlangen Germany to Max and Ida Noether. She was raised as a typical middle class German daughter and went to school from age seven to fifteen. She pursued further study in French and English. By the age of eighteen she passed the examination of the state of Bavaria for teachers of English and French at schools for girls. Emmy was not satisfied ending her education so she decided to try and continue her education.

Emmy had to struggle to obtain the higher education that she wanted. Starting in 1900 women were allowed to enroll in universities only as auditors. Following in her father's footsteps she decided to study mathematics. From 1900 to 1902 Emmy was an auditor for mathematics classes at Erlangen while she studied for the Absolutorium (high school certification). One major problem for women during this time was they were not able to receive the proper education needed to enter college. In 1903 Emmy got her certification to enroll as an auditor at Gottingen and was able to attend lectured by Otto Blumenthal, David Hilbert, Felix Klein, and Hermann Minkowski. She soon returned to Erlangen so she would be able to be a regular student. In December of 1907 Emmy earned her Ph.D. in mathematics. She wrote her thesis under Paul Gordan on complete systems of invariants of ternary biquadric forms. Gordan inspired Emmy in studies of mathematics. Gordon was a strong believer in being straightforward in the writings of mathematics. In his papers a reader could go pages before reading any text.

For the next eight years Emmy researched and substituted for her dad at Erlangen. Emmy did not have any opportunities after she received her doctorate. Women were not allowed to teach in universities so Emmy could not make a living. Her dad allowed her to substitute under his name but she still could not make a salary.

After working with her father, she started to become well known and began working with renowned mathematicians. In 1909 Emmy joined the German Mathematical Association and gave her first public talk establishing herself as a mathematician. From 1910 to 1919 Ernst Fischer had a greater influence then anyone on Emmy's mathematics. Together they studied finite rational and integral bases. In 1916 she derived the conservation laws of physics that made her well known in the physics realm. Hilbert and Klein invited Emmy back to Gottingen in 1915. Emmy's attempt to obtain Habilitation (permission to lecture) in 1915 was denied. Hilbert allowed her to lecture under his name until 1919 when women were granted habilitation. Unfortunately for Emmy she was not given a salary for the work that she did. In 1922 she was recommended for associate professor without tenure. With this promotion she received a small salary. From 1924-1925 B.L. van der Waerden became one of Emmy's students.

As Emmy's career continued she obtained students who loved her studies and managed to help her as well. From 1924-1925 B.L. van der Waerden became one of Emmy's students. He was the popularizer of her work. Emmy's lectures were meant for small groups because she was not a very good public speaker. Van der Waerden made things clearer so individuals could understand her work better. In 1927 Max Deuring became her pupil during her work of non-commutative algebra and hypercomplex areas.

Emmy's career started ending once Germany started having its political struggles. In 1932, after the Nazis took over, she lost her job. In addition to losing her job, the Gottingen School of Algebra, which had been founded by Emmy, was destroyed. She then moved to the United States and in August of 1933 where she started giving lectures at Bryn Mawr and weekly lectures at the Institute for the Advanced Study in Princeton. During her second year she suddenly died after and operation for removal of a tumor.

Emmy Noether was an astounding mathematician. In 1932 Emmy was awarded the Alfred Ackermann-Teubner Memorial Prize for the Advancement of the Mathematical Sciences. She worked hard to find simpler ways of understanding algebra. Unfortunately, Emmy's lectures were not made for large groups. Her delivery of her work was poor, hurried, and inconsistent.

Emmy Noether overcame a lot of prejudice to become the well-known mathematician that she was. As a woman she was held back in her studies as well as her right to lecture. She was very fortunate to have friends like Hilbert that helped her overcome these obstacles. She was also passed up for the Gottingen Academy of Sciences, which several of her colleagues were elected for. Also as a Jew she faced many prejudices. She was kicked out of Gottingen and forced to move to the United States because of her religious beliefs.

"Emmy Noether was an amazing mathematician. She taught us how to think in simple and thus general terms." "She therefore opened a path to the discovery of algebraic regularities where before the regularities had been obscured by complicated specific conditions."(Kimberling, 143). Her greatest talent was toward general mathematical conceptions.

The mathematics of Emmy Noether

The aspect of Noether's work that we will be discussing is Noetherian Rings on Ideals. Her work on Ideals began around 1919-1920. This is the main work that many mathematicians know of Noether. But before we can begin talking about Rings and Ideals, we must first learn some terminology that will guide us through this material. In order to guide you through the several steps we will take an easy example that can be understood by all levels of mathematicians. We are going to take you through the steps of learning about a Noetherian ring by using the integers as our example. There are many examples that we can use, such as matrices, but the integers is the easiest to explain.

- First you must find a set. A set is a collection of objects. The objects are either in the set or are not. Example: {Z} is the set of all integers

- After picking the set we must establish if this set is a group. A group is a structure consisting of a set G and any binary operation on G. We will use + as our binary operation and G= {Z},all integers, as our set. For + to be our binary operation, Z must be closed under addition. For all a,b in Z, a+b must be in Z. Once we have picked our set and the binary operation we must check the following three conditions and see if they hold in order to determine if the set and binary operation make a group.

  - If a,b,c are elements of G, then a+(b+c)=(a+b)+c which is the Associative Property. An example of this property using 2,3,4 which are elements in {Z} is 2+(3+4)=(2+3)+4 which is true for all {Z}. This is just one example but you can infer from this that this property will hold for all Z.

  - There is an element of G, called e, such that for each a element of G, e+a=a; e is a left neutral element for G. This is the Identity Property. An example of this property is e=0 for {Z}, 1+0=1, 2+0=2. This is just one example but you can see how you can use any integer and this would hold true.

  - For each a element of G there is an element b in G such that b+a=e; b is a left inverse of a with respect to e. This is called the Additive Inverse Property. An example of this property is as follows. 2 and -2 are in {Z} -2+2=0=e. This is just one example but notice it would be true for any integer you used.

You have now been given the insight to show why the integers are a group. Now the next step is to show that this group is abelian.

- The next step is to show that the group integers is abelian. For each a and b in G a+b=b+a for it to be an Abelian group. This is the Commutative Property. An example of this property is as follows 2,3 are in {Z}, 2+3=3+2. This is just one example but notice it would be true for any integer you put into this property.

From this information we have shown that {Z} is an Abelian Group under +. Now we must show that this abelian group is a ring.

- The next step is to identify if an abelian group is a ring. A ring is a set R on which are defined by two binary operations + and * (in that order). In order to use multiplication as our binary operation, Z must also be closed under multiplication. For all a,b in Z, (a)(b) must be in Z. With these two operations we need to show that the following three properties hold in order to prove that an abelian group is a ring. In our example we will still be using {Z} as our set.

    - R is an Abelian group under +. The netural element of this group is 0. From the information above we have shown {Z} to follow this rule.

    - Multiplication is associative. Example: 2,3,4 are in {Z}, 2(3*4)=(2*3)4. This is just one example but notice it would be true for any element Z.

    - Multiplication is distributive over addition; for all a,b,c in R a(b+c)=ab+ac and (b+c)a=ba+ca. Example: 2,3,4 are in {Z}, 2(3+4)=(2)(3)+(2)(4) and (3+4)2=(3)(2)+(4)(2). This is just one example but if you put in any integer you will notice that it holds true.

From the information above we have given the intuition that {Z} is a Ring. Now we need to explain what a subset and an Ideal are.

- A is a proper subset of B if and only if all of set A is in set B and A does not equal B. Example: {2Z}, all even integers, is a subset of {Z} because all of {2Z} is in {Z}.

- A nonempty subset I of a ring R is an ideal in R if and only if : (use {2Z} as Ideal.) The following two conditions must hold true in order for the ring to be an ideal.

    - For all a,b in I, a-b is in I. This is equivalent to the fact that I is a subring of R. For an example to see if 2Z is a subring of R we could go back through all the steps in the definition of a ring , but it is a lot easier to show that above condition. In this case if you take two integers and subtract them you will still get an even integer. (4 -2=2)

    - For each y in R and each a in I, ya is in I. For instance with I=2Z and R=Z 2*3=6, where 6 is even and an element of Z. A counter example would be if I=2Z+1 and R=Z then 3*2=6 but 6 is not in I because I includes only the odd integers.

Now that we have found out what an ideal is in a ring we need to look to see what an ACC of Ideals is.

- The ACC of Ideals says that there are infinitely many ideals that get bigger but they are a descending chain of ideals. In order for the ACC of Ideals to be noetherian it must have a stopping point. For example, with the even integers it stops at 2Z and 2Z contains 4Z which contains 8Z and so forth.

- A Notherian Ring is a ring where there is no infinite increasing chain of Ideals in R. We will give you and idea of why it is also a Noetheian Ring. Similar ideas to the properties of ideals we will show that the ideals are even multiples of {2Z}. Now we will give you an idea why other subster of 2Zare not ideals. Now take {2Z+1} ,or the odd integers, as our subset. From the definitions above we can see that {2Z+1} is a subset of {Z}. But for it to be an Ideal we must be able to take any other number in {Z} and be able to multiply it by an element in {2Z+1}, and the answer be a number in {2Z+1}. We can easily show this does not work. An example is that 3 is an element of {2Z+1} and 2 is an element of {Z}. For {2Z+1} to be an Ideal (2)(3)=an odd integer. But we all know that (2)(3)=6 which is not an odd integer. Therefore {2Z+1} is not an Ideal and can't be used in a Notherian Ring because it violates the Ascending Chain Condition. {2Z} is the largest Ideal in {Z}. {2Z} is an Ideal that we showed above. Now take {4Z}. it is contained in {Z} and in {2Z}. Just like this, {2Z} contains {4Z} contains {8Z} and etc. But as you can see the Ideals contain even integers but the number of integers in each Ideal keeps getting smaller than the one before it. So the largest Ideal we can get is {2Z}. This shows that there is a finite stopping place for the Ideals. Which means that {Z} is a Notherian Ring. (All definitions courtesy Landin and also Dummit).

Now we will, briefly, show a ring which is not Noetherian. This example is based on all continuous functions on the closed interval from 0 - 1. Lets assume we have a set defined as follows f:[0,1] to the Reals with (f+g)(x)=f(x)+g(x) and (f*g)(x)=f(x)*g(x). From the information provided above we can show that f:[0,1] to the reals is a ring. We can then also show that this ring is not Noetherian. This is true because it violates the Ascending Chain Condition. In other words, In={f in F such that f (1/n) = 0 for all n greater than or equal to N}. This shows that f is an element of I n+1 such that f is not an element of I n. So all this just says that as the Ideals of the ring get larger, they never have a stopping place. This is the easiest way to show that a ring is not Noetherian.

Now we will discuss the importance of Emmy Noethers' work and how it pertains to her, mathematics and the world around us. The work that she did with Ideals is one of her biggest claims to fame as far as mathematicians are concerned. This phrase from "In Memory of Emmy Noether" gives us a nice example of the way that Noether related her math to the people and world around her and to the improtance that her work had. "I believe that, of everything that Noether did, it is the foundations of general Ideal theory and everything connected with this that has had, is continuing to have, and will have in the future the greatest impact on mathematics as a whole"(158). From the work she did, she opened up whole new areas of math for future generations to flourish in. "Noetherian Rings are one of the basic structures of all of mathematics"(Ratliff,169). The definition of a Noetherian Rings can be modified to form the definition of a Noetherian Modules which are used in vector spaces. With all of us being math majors, we know how important vector spaces can be. Noetherian rings also have physics applications. Noether made great strides to show that simple was better. "She thought that the easiest way to find an answer to a complicated algebraic guestion was to use simple and general algebraic concepts instead of cumbersome computations to do so."(158) We think that this gives justice to the claim that Emmy Noether was the Mother of Algebra.

Through discrimination Emmy Noether fought to become one of the greatest mathematicians of all time. With her work in Algebra, she opened the door for new explorations in both math and the sciences. Without this woman, todays world would be a different place and one without many of the advantages that this womans' work brought to us. She fought a long battle to become a great mathematician, she fought a long battle and won.