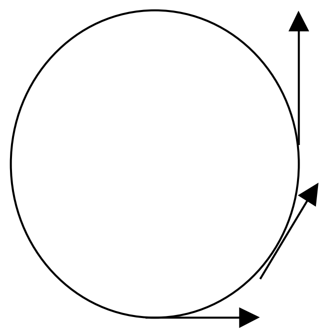
Alice's Math

The tangent is defined as $F'(x)$ of a parameterized curve. If we were given $F(x)=(a\cos(t), a\sin(t), bt)$ where t is an element of R. The tangent vector is $F'(x)=(-a\sin(t),a\cos(t),b)$, note that we have taken the derivative with respect to t. Using this same example we can find the normal vector. The normal vector is perpendicular to the tangent vector and is defined as $F''(x)$. So with the last example the normal vector would be $F''(x)=(-a\cos(t),-a\sin(t), 0)$.

The length of the normal is determined by the sharpness of the curve. The sharper the curve the longer the normal or as the curve becomes more like a line the normal shortens. A real life application of this is driving your car on a curvy mountain road. Think about when you drive around sharp turns and long drawn out turns. The force or the normal which pulls you inward is stronger on the sharp turn then it is on the long drawn out curve.

Recall that a plane is built up of at least two vectors. So the tangent vector and the normal vector build up a plane. So as the graph of a curve grows the plane (built from the tangent vector and the normal vector) moves or oscillates with the graph. Let’s think about simple surfaces to start this idea.

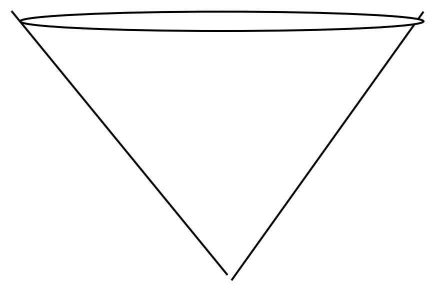
The first example we will use is a circle. Look at the picture and see the planes formed.



As seen in the picture the tangent and the normal changes direction with the surface of the circle. As a result of the change the plane also has to change. This is what we describe as the oscillating plane. Note that the circle is a “nice” surface. Meaning that it has no sharp points or edges. Because there are no edges the plane can oscillate smoothly with the path of the surface.

Not all surfaces are “nice” though. So let’s look at one that has some sort of edge or point.

Look at this picture of a cone and try to see what happens with the plane at its point.



As you can see at the point of the cone there is infinitely many tangents. As a result there are infinitely many planes. To stop confusion we just say that the plane is undefined at such a point. To understand this more think about Calculus. Remember that you can't take a derivative of a point or sharp edge on a graph. So in this case again you cannot define a tangent which leads to an undefined plane.