Doris Wood Schattschneider

Doris Schattschneider received her M.A. and PhD in mathematics from Yale University with a dissertation on "Restricted roots of a semi-simple algebraic group." She is currently a Professor of Mathematics at Moravian College in Bethlehem, Pennsylvania and has taught for over thirty years. She is a woman of great accomplishments; such as, the first woman Editor of Mathematics Magazine (1981-1985), and she was also the first woman to deliver Pi Mu Epsilon's J. Sutherland Frame Lecture (1988). Schattschneider is internationally known for her work with tessellations of the plane and her exposition of Dutch artist M.C. Escher.

Born on October 19,1939 in New York, Schattschneider was the second of four children to Robert and Charlotte Wood. Charlotte Wood received her master's degree in classics from Cornell University and taught Regents Latin in high schools and is a coauthor of a widely used Latin textbook. Robert Wood graduated from City College of New York and was a mechanical engineer for the Bureau of Bridges of the City of New York. Born a daughter of two professional parents, there was no question that Doris and her siblings would be fully educated and that family expectations would be very high. For

example, during Doris's youth, the Wood household revolved around lively dinner table discussions in which her father always wanted to know what had happened that day in school. If anyone would ask a question about the meaning of a word or topic, the questioner had to leave the table and look up the answer before dinner could proceed. Thus, Doris and her siblings were introduced to research skills at the dinner table.

Mathematics came easily to Doris, and she enjoyed its challenges. In high school she tutored her peers for the mathematics portion of the Regents examinations. Teaching was in her blood; not only because her mother was a teacher, but her grandfather had been a schoolteacher as well. She was encouraged by her high school mathematics teacher; a woman who tantalized her with the suggestion that some questions not answered by algebra would be answered when she learned calculus in college. In college she enjoyed physics, but that instructor did not encourage her to pursue it. Studio art and sculpture were also her favorite activities. Doris wanted to create work rather than analyze the works of others. She finally decided to study mathematics and received her A.B. degree from the University of Rochester.

Doris Schattschneider worked on many aspects of mathematics. Symmetry and geometric models have long held a special fascination with her. Doris was interested in both geometry and art and this led naturally to the study of tiling problems and the work of the Dutch artist M.C. Escher. She has written many scholarly articles that have dealt with many subjects within mathematics. Here we will concentrate on her work in taxicab geometry.

Taxicab geometry is a field of geometry that differs greatly from the Euclidean geometry that most of us are used to. In taxicab geometry the only way that you can

move between two points is to move in a horizontal or vertical straight line or turn at a right angle. Think of a taxicab driver on a city grid. The driver can only follow the streets. He must either go straight or turn left or right (at a right angle.) This is how taxicab geometry got its name. This differs from Euclidean geometry in that in Euclidean geometry you connect any two points with a straight line. Recall the distance equation from Euclidean geometry. The distance between any two points (x_1, x_2), (y_1, y_2) is given by

$$D = ((x_2 - x_1)^2 + (y_2 - y_1)^2)^{1/2}.$$

In taxicab geometry the distance equation changes. The distance between those same two points in taxicab geometry is given by
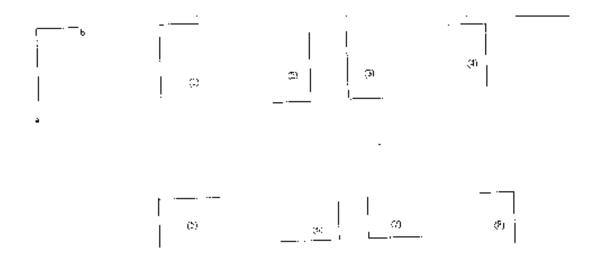
$$D_T = \text{absolute value of } (x_2 - x_1) + \text{absolute value of } (y_2 - y_1).$$

This is known as the taxicab metric. The taxicab metric was first discovered by Hermann Minkowski (1864-1909) as a special case of a metric defined in terms of an arbitrary convex set centered at the origin. Another interesting fact about taxicab geometry is that Euclid's fourth postulate, side angle side does not hold in taxicab geometry. (I could not find the formal proof of that outcome; however every source I consulted made that statement.)

This new definition of distance tends to make one wonder where taxicab geometry is used. Other than the most obvious case with a taxi driver and a city, taxicab geometry can be used anytime to find distance when traveling in a straight line is not possible. Also biologists have found the metric D_T useful in the measurement of "niche overlap" and notion of ecological distance between species.

For this paper we will look at Schattschneider's paper entitled "The Taxicab Group." In the paper Schattschneider addresses the problem of "given a space S, endowed with a metric d, describe the group G of isometries with respect to the metric d." The metric d she was referring to was the taxicab metric that was explained earlier. First, an isometry is a translation, rotation, reflection or glide reflection that preserves distance between two points in the plane. To answer this question Schattschneider first made some geometric observations. The first observation is that if x and y are points in the plane, then the taxicab distance is just the length of the L shaped figure that joins them, where the sides of L are parallel to the coordinate axes. Because of this observation we can immediately rule out Euclidean isometries that map horizontal or vertical lines to lines that are not parallel to the coordinate axes; for doing this would increase taxicab distance. Also note, that if x and y lie on a straight line then their Euclidean distance is equal to their taxicab distance.

After making this observation, Schattschneider sates that there are only eight types of Euclidean isometries that preserve distance that we must consider. The eight isometries are as follows: (1) translation, (2) rotation of 180°, (3) glide reflection in horizontal line, (4) glide reflection in vertical line, (5) glide reflection in line with slope -1, (6) glide reflection in line with slope 1, (7) rotation of 90°, and (8) rotation of 270°. The number in ( ) refers to the figure on the next page. The figure is numbered 1 through 4 on the top left to right and 5-8 on the bottom left to right.

After observing that these eight isometries in Euclidean geometry preserve distance in taxicab geometry, Schattschneider goes on to ask the question do anymore exist? Schattschneider then goes through a rather complicated proof in which she blends geometry and algebra to arrive at her conclusion. We won't go through that proof here, however Schattschneider did indeed prove that the eight that she observed first were the only eight that existed.

Over the years Schattschneider has authored many scholarly articles on tiling the plane and has produced three activity books. She was the first vice president of the Mathematical Association of America (MAA). In 1993 she received the (MAA) award for Distinguished Teaching of College or University Mathematics. She is also involved with The American Mathematical Society; the Association for Women in Mathematics, The National Council of Teachers of Mathematics, and The Association of Teachers of Mathematics. She is also a member of Phi Beta Kappa and Pi Mu Epsilon.

References:

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Notable Women in Mathematics pp 214-218

These are good sources.   In each one we found something that we had not found before.
It was hard to find sources, but these that we found are excellent.