**Nathaniel Dean**

Nathaniel Dean is considered one of the greatest mathematicians in graph theory. Not much is known about his life except for his education and publications. Dean obtained his B>S. in Mathematics from Mississippi State University (Williams, 1). He has always valued education and proved this by obtaining his M.S. Degree in Applied Mathematics and his Ph.D. from Vanderbilt University in 1987 (1). Dean wrote his doctoral dissertation on "Contractible Edges and Conjectures and Path and Cycle Numbers". Dean;s main focus of study is Graph Theory, but he also has done research i Algorithms, Geometry, and Combinatories (Dean, 1).

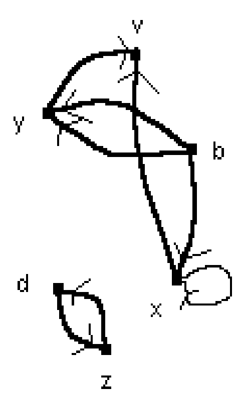
Dean has had a great deal of successes in his career, which began upon his graduation from Vanderbilt. From 1987 to 1998, Dean was a member of the Software Production Research Department of Bell Laboratories (Williams, 1). It was also during this time period that Dean married Rhacel Sabalande on February 14, 1996 (Dean, 1). He went on to have three sons with her who all excelled in their academics (1), after Dean left Bell Laboratories; he joined the staff at Rice University as an associate professor in the department of Computational and Applied Mathematics (Williams, 1). From there he joined on at Brown College as a faculty associate in April 2000 (1). Dean has also participated in the PBS series “Life By the Numbers” and has co-authored tow books (Dean, 1). He has also published 45 papers: 26 in mathematics and 19 in computer science (Williams, 1).

Dean is not only interested in furthering his knowledge of mathematics, but is also very active in the education of minorities. He has several links on his website to historically black universities and he has memberships in several advancement of African-American education organizations (Dean, 1). He wants to make sure that minorities have the same, if not better, opportunities that he had in obtaining an excellent education.

Dean wrote a paper called Gallai’s conjecture for disconnected graphs. In this paper we are going to be looking into the idea of graph theory. In this paper he wrote on simple, finite, undirected graphs. A graph is a mathematical abstraction that is useful for solving many kinds of problems. Graph theory is the study of relations on finite sets which can be visualized with dots and lines. A perfect example would be to show how every PC is somehow connected to other PCs. A PC would be a vertex and then every vertex is connected through the interent. Their connection would be the edges. It is important to study these connections because this type of system needs to be checked in terms of vulnerability. For example by chance a computer loses its connection to the Internet we need to know if everyone will lose his or her connection.

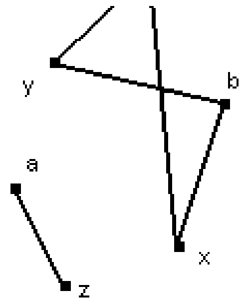
A graph consists of vertices and edges. Edges connect two vertices. A graph is a pair (V,E) where V is a finite set of vertices and E is the edges formed between two vertices.

Dean talks about undirected graphs therefore we need to understand what one looks like. In a directed graph, edges are ordered pairs, connecting a source vertex to a target vertex. Here is an example of a directed graph.




This graph is a graph because it is made up of vertices and edges. It is a directed graph for several reasons. One reason this graph is directed is because the edges have direction. Another reason this graph is directed is because between the vertices z and d there is a loop. A loop is where two edges are between two vertices and their direction is opposite of each other. The final reason that this graph is directed is because of the edge found at vertex x where it starts and ends on the vertex x. This is called a loop.

In an undirected graph, edges are unordered pairs and connect the two vertices in both directions. Here is an example of an undirected graph.




This is a graph because it is made up of vertices and edges. This is an undirected graph because there are no directions on the edges and there are no loops.

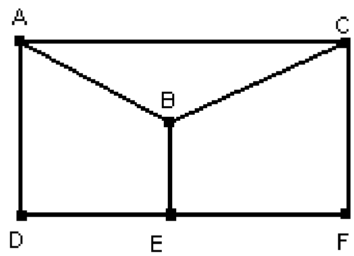
You can see the similarities and differences between the two graphs. The two graphs are similar because they have the same vertices and same edges. There are several differences between the two graphs. In the directed graph the edges have directions versus the undirected do not have directions. Also in the second graph (a,z) is the same thing as (z,a) therefore the edges between them are collapsed into one line. This has to do with the binary operations. When there are two vertices then they output one edge. Finally looking at the first graph you see there are two lines (b,y) and (b,y) these are collapsed into one line in the second graph. Dean wrote on the undirected type of graph.

The next characteristic that we need to understand is what a simple graph is. A simple graph can be thought of as G=(V,E,I). V and E a finite sets of the vertices and edges respectively. I is an incidence relation saying that every element of E is incident with exactly two distinct elements of V and not two elements of E are incident to the same pair of elements in V. This means that every edge only has two vertices and there is only one edge for every two vertices.

The final characteristic of a graph Deal deals with is being finite. A finite graph is one that has a set number of vertices and edges.

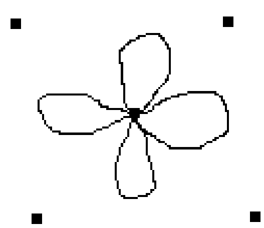
Other definitions that are important to our understanding graph theory are as follows.

a) The degree of a vertex is the number of edge ends at that vertex.



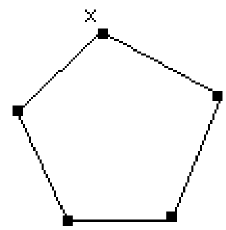

In the example above points A, B, C, and E have a degree of 3 and points D and F have a degree of 2. This graph is also undirected since the edges do not have direction. Also this graph is simple since there is a finite amount edges and vertices. As you notice though there is not an edge between vertices A and F. It is not necessary for an edge to be between every vertex.

b) A graph is connected if there is a path connecting every pair of vertices.

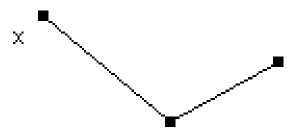


In this example the graph is disconnected and made up of five components. The five components are as follows. The four outside points are each a component and then the point with the four loops is a component.

c) A cycle is a graph that starts and stops at the same point.

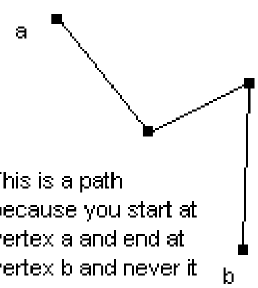



This graph is a cycle becaues it begins and ends at point x.




This graph is not a cycle because the graph begins at x but does not end at x.

d) A path is a pathway that starts at one points and ends at another and does not double back over any of the same edges.

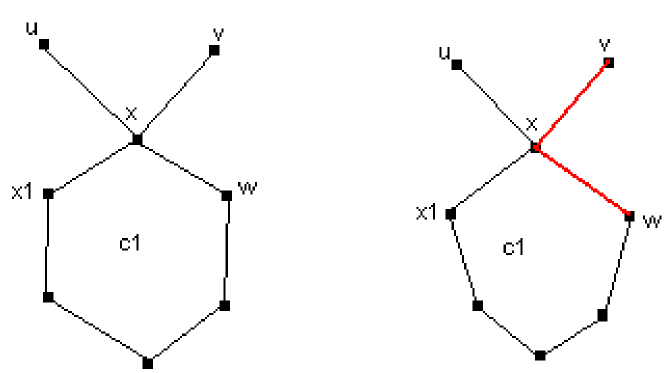



This is a path because you start at vertex a and end at vertex b and never it never crosses back over and edge already formed

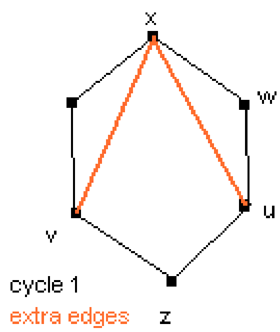
In Dean's paper he proved Lemma 2.1 which says that if you "let $G_r$ be a graph decomposable into r cycles and at most two edges, all containing the vertex x, for r=1,2,3. Then $p(G_r)=r+1$." p(Gr) is the number of paths it takes to decompose a graph. Dean is saying that if there are 3 cycles that it takes 4 paths in order to decompose the graph. He proves this by contradiction. I am going to explain his proof in this paper.

The proof starts out “assume that G is a counter example. Call the cycles C1, C2, C3. By adding extra pendent edges at x we can assume that exactly two edges xu, xv are used in the decomposition. Let xxiE(Ci) for I=1,2,3.” In the end Dean will find out that G is not a counter example and therefore prove that the lemma is correct. He ends up proving that G is an example by finding that according to how many cycles there are it takes one more path than the number of cycles present. His basic concept is that if he is able to take one side out of each cycle he is able to prove that there are r+1 paths that decompose the graph.

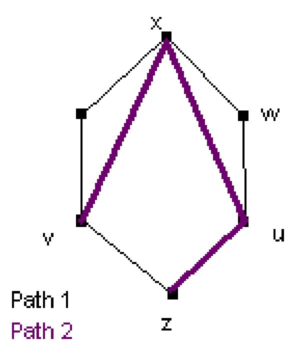
He then starts out with supposing r=1. He says “Let xw$\in$E(C1), then the paths wxv, C1-xw+xu decompose G1. The following graph shows what I have just stated.




The graph on the left is just a representation of what the graph looks like. The graph on the right shows the two paths C1-xw+ux and vxw are possible. Since we have two paths we have just proved that if there is one cycle then it takes two paths to decompose it, which proves that G is not a counter example. Therefore we need to try a new situation and assume that u and v are elements of C1. Then the graph looks like the following.

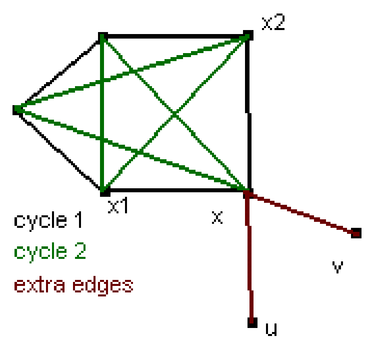



This graph is possible since the lemma states there are two other edges and not necessarily two other vertices. The proof goes on to say that there must be a point zv such that uz is an edge of C1. This edge is needed so that we have an edge of the cycle that we can take out and use to make the second path. The paths that can be formed look like the following graph.

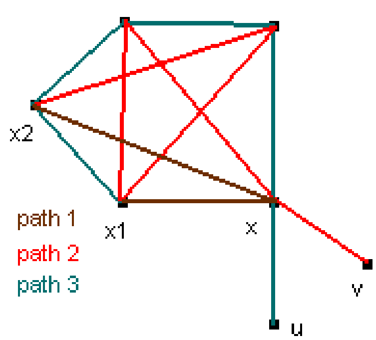



We have just done a direct proof showing that if there is one cycle then it takes two paths that decompose it. We first looked to see if u, v$\notin$ V(C1) and then if they were elements of V(C1). Both ways we see that we find two paths to decompose the graph. We were trying to prove that it would take less then two paths or more then two paths to decompose G.

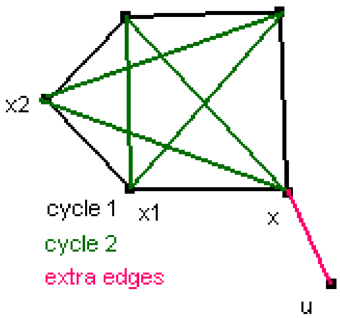
The next case we look at is if r=2. This gets a lot more complicated because of the two graphs. The first way we try to prove is by saying $u \notin V(C1)$ and $v \notin V(C2)$. Then we get the following graph.



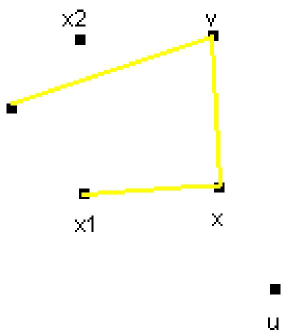

From here we can easily find three paths that make up this graph. The paths are c1-x1x+xu, c2-x2x+xv, x2xx1. The following graph shows the paths.



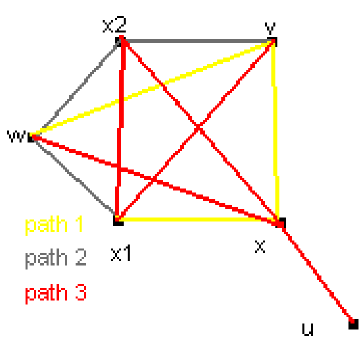

The path x1xx2 takes a side out of each cycle and then from there it is easy to find two more paths. This follows the lemma that p(Gr)= r+1 therefore we have still not proved that G is a counter example. Now we must try the idea that maybe $v \notin V(C2)$.



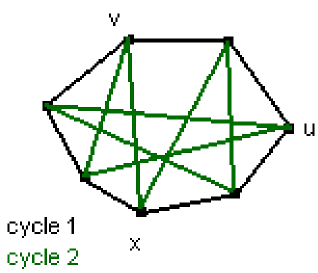

Now Dean states there must be a point w≠ x1



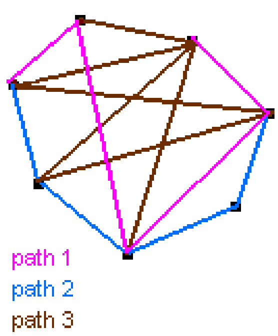

In the graph above we see the path x1xvw. The edge xv is in cycle 1 and the edge vw is in cycle 2 therefore we have just taken an edge from each cycle out and the following graph shows the three paths that are possible to form. The paths are c1-vx-xx1, c2-wv+xu, wvxx1.




The final possibility that we need to try is that if v,u∈ V(C1) and V(C2). If they are the graph looks like the following picture. If you notice the cycles can start and end at any vertex.




Since we have this graph we know that there is one vertex adjacent to v that is not v and one vertex next to u that is not v because there are two adjacent vertices to both points. Therefore there is a vertex adjacent to u that is in C1 called u1 and there is a point v1 that is adjacent to v that is in C2. With these new points we can make a path u1uxvv1 that contains an edge from both cycles and therefore we can decompose the graph with three paths. The following paths are c1-v2vu1u, c2-uxv+vu1, v2vxuu1. The following graphs shows what the paths look like.




The final case that we are going to look at is if r=3. It is starting to get more difficult because of the counting. What Dean did was start by saying that u∉ V(i) where i=1,2,3. Then all you have to notice is if you take once side out of each cycle you can decompose the graph using four paths. This contradicts the fact that graph G is a contradiction to the lemma. The proof continues on with the counting getting more and more difficult but he continues to take one side out of each cycle and is able to decompose into the correct amount of paths. He stops at three cycles because the lemma is only trying to prove it up to three cycles.