REVIEW EXERCISES FOR CHAPTER 7

True-false: For Exercises 1–5, determine if the statement is true or false. If it is true, explain why. If it is false, provide a counterexample or an explanation.

1. In a first-order method, halving the step size typically halves the error.

2. In a second-order method, doubling the number of steps usually doubles the accuracy.

3. In a fourth-order method, doubling the number of steps usually quadruples the accuracy.

4. Improved Euler's method is a second-order method.

5. Runge-Kutta is a third-order method.

6. Consider the initial-value problem

$$\frac{dy}{dt} = t - y^3, \quad y(0) = 1$$

over the interval $0 \le t \le 1$.

(a) Calculate the Euler approximation to the solution using $n = 4$ steps and draw its graph.

(b) Calculate the improved Euler approximation to the solution using $n = 4$ steps and draw its graph.

(c) Calculate the Runge-Kutta approximation to the solution using $n = 4$ steps and draw its graph.

(d) Using a calculator or computer, repeat parts (a)–(c) using $n = 100$ steps.

7. Consider the competing species system

$$\frac{dx}{dt} = 2x - x^2 - xy$$
$$\frac{dy}{dt} = 3y - y^2 - 2xy$$

and the initial condition $(x_0, y_0) = (3, 3)$.

(a) Using a calculator or computer, calculate the Euler approximation to the solution over the interval $0 \le t \le 5$ using $\Delta t = 0.1$. Plot the result in the xy-phase plane and plot the corresponding $x(t)$- and $y(t)$-graphs.

(b) Repeat part (a) using improved Euler's method.

(c) Repeat part (a) using Runge-Kutta.