

Lab 1.1 Logistic Population Models with Harvesting (pg. 133)

In this lab, we consider logistic models of population growth that have been modified to include terms that account for harvesting. In particular, you should imagine a fish population subject to various degrees and types of fishing. The differential equation models are given below. (Use the values of the parameters k , N , a_1 , and a_2 listed in Table 1 in the line corresponding to the last nonzero digit in your phone number.) In your report, you should include a discussion of the meaning of each variable and parameter and an explanation of why the equation is written the way it is.

We have discussed three general approaches that can be employed to study a differential equation: Numerical techniques yield graphs of approximate solutions, geometric/qualitative techniques provide predictions of the long-term behavior of the solution and in special cases analytic techniques provide explicit formulas for the solution. In your report, you should employ as many of these techniques as is appropriate to help understand the models, and you should consider the following equations:

1. (Logistic growth with constant harvesting) The equation

$$\frac{dp}{dt} = kp \left(1 - \frac{p}{N}\right) - a \quad (1)$$

represents a logistic model of population growth with constant harvesting at a rate a . For $a = a_1$, what will happen to the fish population for various initial conditions? (Note: This equation is autonomous, so you can take advantage of the special techniques that are available for autonomous equations.)

2. (Logistic growth with periodic harvesting) The equation

$$\frac{dp}{dt} = kp \left(1 - \frac{p}{N}\right) - a(1 + \sin(bt)) \quad (2)$$

is a nonautonomous equation that considers periodic harvesting. What do the parameters a and b represent? Let $b = 1$. If $a = a_1$, what will happen to the fish population for various initial conditions?

3. Consider Equation 2, but now let $a = a_2$. What will happen to the fish population for various initial conditions with this value of a ?

Your report: In your report you should address these three questions, one at a time, in the form of short paragraphs. Begin Questions 1 and 2 with a description of the meaning of each of the variables and parameters and an explanation of why the differential equation is the way it is. You should include pictures and graphs of data and of solutions of your models as appropriate. (Remember that one carefully chosen picture can be worth a thousand words, but a thousand pictures aren't worth anything.)

Table 1: Possible choices for the parameters

Choice	k	N	a_1	a_2
1	0.25	4	0.16	0.25
2	0.50	2	0.21	0.25
3	0.20	5	0.21	0.25
4	0.20	5	0.16	0.25
5	0.25	4	0.09	0.25
6	0.20	5	0.09	0.25
7	0.50	2	0.16	0.25
8	0.20	5	0.24	0.25
9	0.25	4	0.21	0.25

Drawing Slopefields with Maple

1. Load the `DEtools` package to get the `DEplot` command. (Use a colon to suppress listing all the commands.)
`> with(DEtools):`

2. Define the differential equation
`> DiffEq := y'(t) = t2 - y(t)2 + sin(ty(t))`

$$\text{DiffEq} := y'(t) = t^2 - y(t)^2 + \sin(ty(t))$$

3. Set up initial conditions, if desired. Careful with the brackets!
`> Inits := [[y(0) = 0], [y(0) = 1]]`

$$\text{Inits} := [[y(0) = 0], [y(0) = 1]]$$

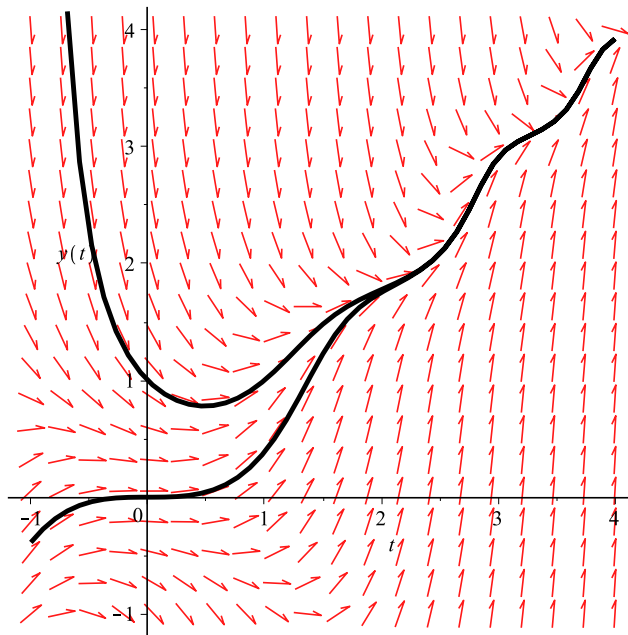
4. Define the variables' ranges and add any options.

`> Ranges := (t = -1..4, y = -1..4):`

`> PlotOptions := linecolor = black:`

5. And plot the slopefield with solution curves.

`> DEplot(DiffEq, [y], Ranges, Inits, PlotOptions)`



(Graphs can be copied and pasted into a word processor document.)

For more information about `DEplot` and to see the possible options, enter

`> ? DEplot`