

Work quickly and carefully, following directions closely. Answer all questions completely.

FOR ALL PROBLEMS: Define P , Q , R , and S to be the four digits in your given number.

$$P = \underline{\quad}, \quad Q = \underline{\quad}, \quad R = \underline{\quad}, \quad S = \underline{\quad}.$$

§I. TRUE and/or FALSE. Circle your answer. There are 2 questions at 2 points each.

- TRUE or FALSE: Euler's method is a 'predictor-corrector' numerical method.
- TRUE or FALSE: The Runge-Kutta method is based on $\begin{cases} \text{Simpson's Rule of integration} & (P \text{ is even}) \\ \text{the Trapezoid Rule of integration} & (P \text{ is odd}) \end{cases}$.

§II. MULTIPLE CHOICE. Circle your answer. There are 2 question at 5 points each.

- The order of the Runge-Kutta numerical method we studied is
 - $\mathcal{O}(h)$
 - $\mathcal{O}(h^2)$
 - $\mathcal{O}(h^3)$
 - $\mathcal{O}(h^4)$
- For Heun's method, the new y value is $y + \Delta y$ where Δy is given by:
 - $\Delta y = f(t, y) \cdot h$
 - $\Delta y = \frac{1}{2}(m_L + m_R)h$
 - $\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$
 - none of the above
 - all of the above

§III. PROBLEMS. *You must show your work to receive credit.* There are 3 problems at 10 points each.

- Suppose that the overall error in using Heun's method on $y' = f(t, y)$ is $\epsilon \leq 10^{-3}$ for a given stepsize h . What stepsize h do we need to use to achieve an error of $\epsilon \leq 10^{-6}$?



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2. Consider the initial value problem $y'' - Py' + Qy = 0$ with $y(0) = R$ and $y'(0) = S$. Do one step of Euler's method to find $y_1 \approx y(0.1)$ using a stepsize $h = 0.1$.

3. Why would an applied mathematician use a numerical method instead of just solving an initial value problem symbolically to get an exact solution?



What is like *kryptonite* to moms?