MAT 3130	Test 1	NAME:__________________
FALL '13	Form I	EMAIL ID:__________________

Work quickly and carefully, following directions closely. Answer all questions completely.

FOR ALL PROBLEMS: Define P, Q, R, and S to be the four digits in your given number.

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§I. TRUE and/or FALSE. Circle your answer. There are 2 questions at 2 points each.

1. TRUE or FALSE: The differential equation $dy/dt = \cos(Pt) + y$ is $\begin{cases} \textit{separable} & \text{your number } Q \text{ is even} \\ \textit{linear} & \text{your number } Q \text{ is odd} \end{cases}$.

2. TRUE or FALSE: A unique solution must exist for the differential equation $dy/dt = \sqrt{ty}$ with $y(R) = 0$ where R is your number.

§II. MULTIPLE CHOICE. Circle your answer. There are 2 question at 5 points each.

1. The correct integrating factor μ for the differential equation $y' = t^2 - y + S$ where S is your number is

   (a) $\mu(t) = e^{-t}$ (b) $\mu(t) = e^{St}$ (c) $\mu(t) = e^{t^2}$

   (d) none of the above (e) all of the above

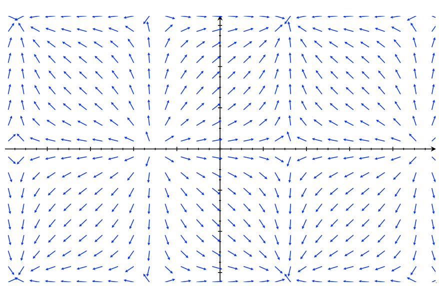
2. The differential equation $\dfrac{dy}{dt} = \dfrac{y}{t^2+1} + \dfrac{y}{t}$ is

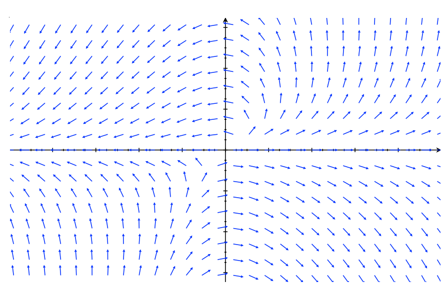
   (a) separable, not linear (b) linear, not separable (c) both separable and linear

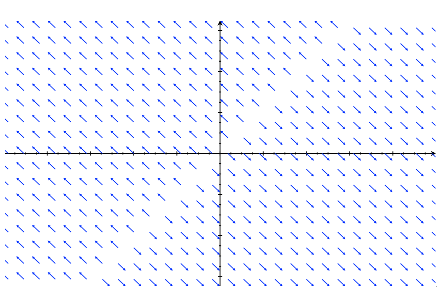
   (d) none of the above

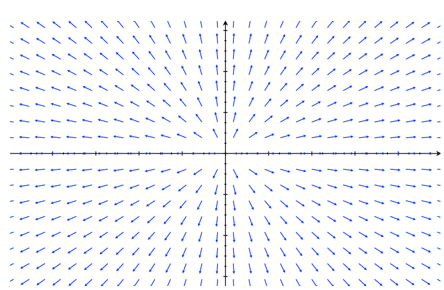
§III. MATCHING. Match the system of differential equations to its corresponding direction field by writing the equation's number below the field's graph. (One direction field has no match.) There are 3 matches at 5 points each.

$$I. \begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} x - y \\ x \cdot y \end{bmatrix} \qquad II. \begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \qquad III. \begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} \cos(x) \\ \sin(y) \end{bmatrix}$$









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§IV. PROBLEMS. *You must show your work to receive credit.* There are 4 problems at 20 points each.

1. Solve the given differential equations:

   (a) $\dfrac{dz}{dx} = \dfrac{x}{z + x^2 z}$ and $z(0) = P$

   (b) $\dfrac{dy}{dt} = y - Qe^{-t}$

2. A cup of coffee is brought into a classroom. The classroom is at 20°C; the coffee is initially $(20 + R) =$ _____°C. Newton's *law of cooling* states that the rate of the coffee cooling is proportional to the difference between the coffee's temperature and the room temperature.

   (a) Write a differential equation model for the temperature T of the coffee.

   (b) Draw a *phase line* showing any equilibria and the solutions behaviour.

(a)

(b)

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3. A *logistic population with a constant harvest* $20h$, (h is the percent of the carrying capacity population 20 that is being harvested) is modeled by

$$\frac{dp}{dt} = \frac{1}{5}p \cdot \left(1 - \frac{p}{20}\right) - 20h$$

   (a) Determine the equilibrium point(s).

   (b) What is the largest value of h (percent harvest) that is sustaineable? I.e., what value of h leaves a positive equilibrium for $p(t)$?

4. Consider the predator-prey system of rabbits r and foxes f: $\begin{bmatrix} r'(t) \\ f'(t) \end{bmatrix} = \begin{bmatrix} 0.4r - 0.05rf \\ -f + 0.05rf \end{bmatrix}$ with the initial value at $t_0 = 0$ of 30 rabbits and S foxes (S is your number).

   (a) What are the two equilibrium points?

   (b) Using *Euler's method* with $\Delta t = 0.1$, find $\begin{bmatrix} r(0.1) \\ f(0.1) \end{bmatrix}$.