MAT 3130	Test 2	NAME:__________
FALL '13	Form II	EMAIL ID:__________

Work quickly and carefully, following directions closely. Answer all questions completely.

FOR ALL PROBLEMS: Define P, Q, R, and S to be the four digits in your given number.

$$P = \_\_\_\_, \quad Q = \_\_\_\_, \quad R = \_\_\_\_, \quad S = \_\_\_\_.$$

§I. TRUE and/or FALSE. Circle your answer. There are 3 questions at 2 points each.

1. TRUE or FALSE: The origin is the only equilibrium point for any linear system.

2. TRUE or FALSE: The x- and y-nullclines of a nonlinear system cannot intersect.

3. TRUE or FALSE: The value of the improper integral $\int_0^\infty \delta(t-R)\, e^{-t}\, dt$ (where R is your number) is e^{-R}.

§II. MULTIPLE CHOICE. Circle your answer. There are 2 question at 5 points each.

1. The *eigenvalues* for the system $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & P \\ 0 & Q \end{bmatrix}$ (where P and Q are your numbers) are $\lambda =$

   (a) 1 and 0 (b) P and Q (c) P and 0 (d) 1 and Q

2. Linearizing the differential equation $\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} x^2 + \sin(x) \\ y - \sin(xy) \end{bmatrix}$ at the origin gives $\frac{d}{dt}\vec{V} = \mathbf{A} \cdot \vec{V}$ with $\mathbf{A}$ equal to:

   (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

§III. MATCHING. Four functions $y(t)$ and twelve functions $Y(s)$ are given below. For each function $y(t)$, match it with its Laplace transform $Y(s)$. There are 4 matches at 3 points each.

$y = e^{2t}\cos(2t)$	$y = e^{2t} - e^{-2t}$	$y = \sin(t - \pi)$	$y = u(t-2) \cdot e^{3t-6}$
$Y =$ ☐	$Y =$ ☐	$Y =$ ☐	$Y =$ ☐

I. $Y = \frac{4}{s^2 - 4}$	II. $Y = \frac{4}{s^2 + 4}$	III. $Y = \frac{2}{s^2 + 4}$
IV. $Y = \frac{2s}{s^2 + 4}$	V. $Y = \frac{s}{s^2 - 4}$	VI. $Y = \frac{-1}{s^2 + 1}$
VII. $Y = \frac{s}{s^2 + 4}$	VIII. $Y = \frac{e^{-2s}}{s + 3}$	IX. $Y = \frac{e^{-2s}}{s - 3}$
X. $Y = \frac{s}{s^2 - 4s + 8}$	XI. $Y = \frac{s - 2}{s^2 - 4s + 8}$	XII. $Y = \frac{e^{-\pi s}}{s^2 + 1}$

☐ *Go on to the next page.*

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§IV. PROBLEMS. *You must show your work to receive credit.* There are 4 problems at 20 points each.

1. The linear differential system $\frac{d}{dt}\vec{Y} = \mathbf{A} \cdot \vec{Y}$ has eigenvalues $\lambda_1 = (-1)^P$ and $\lambda_2 = \left(-\frac{1}{2}\right)^Q$ (where P and Q are your numbers). Determine what type of equilibrium the origin is.

2. Consider the differential equation $x''(t) + x'(t) + Rx(t) = 0$ with $x(0) = 1$ and $x'(0) = 1$. (R is your number.)

   (a) Convert the differential equation to a system of two variables x and v.

   (b) Determine the type of equilibrium of the system at the origin.

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3. Find the inverse Laplace transform of the function $F(s) = e^{-Qs} \cdot \frac{3!}{s^4}$ where Q is your number.

4. Use Laplace transforms to solve the initial value problem $\frac{d^4x}{dt^4} - x = 4$ with $x'''(0) = x''(0) = x'(0) = x(0) = 0$.

☐ EC—TEST JEOPARDY: The answer was "Twelve sided geometric shape," not "Giant blood-thirsty dinosaur." What was the question?