MAT 3130	Spring Project	NAME: ____________
FALL '13		LAST 4 PHONE DIGITS: ____________

PROJECT 5.1 HARD AND SOFT SPRINGS

In this lab, we continue our study of second-order equations by considering "nonlinear springs." In Sections 2.1 and 2.3, we developed the model of a spring based on Hooke's law. Hooke's law asserts that the restoring force of a spring is proportional to its displacement, and this assumption leads to the second-order equation

$$m\frac{d^2y}{dt^2}+ky=0.$$

Since the resulting differential equation is linear, we say that the spring is linear. In this case the restoring force is $-ky$. In addition, we assume that the friction or damping force is proportional to the velocity. The resulting second-order equation is

$$m\frac{d^2y}{dt^2}+b\frac{dy}{dt}+ky=0.$$

Hooke's law is an idealized model that works well for small oscillations. In fact the restoring force of a spring is roughly linear if the displacement of the spring from its equilibrium position is small, but it is generally more accurate to model the restoring force by a cubic of the form $ky+ay^3$, where a is small relative to k. If a is negative, the spring is said to be hard, and if a is positive, the spring is soft. In this lab we consider the behavior of hard and soft springs for particular values of the parameters. (Use parameter value(s) from Table 5.1 in the row corresponding to the last nonzero digit in your phone number.) In your report, you should analyze the phase planes and $y(t)$- and $v(t)$-graphs to describe the long-term behavior of the solutions to the equations:

1. HARD SPRING WITH NO DAMPING. The first equation that you should study is the hard spring with no damping; that is, $b=0$ and $a=a_1$. Examine solutions using both their graphs and the phase plane. Consider the periods of the periodic solutions that have the initial condition $v(0)=0$. Sketch the graph of the period as a function of the initial condition $y(0)$. Is there a minimum period? A maximum period? If so, how do you interpret these extrema?
2. HARD SPRING WITH DAMPING. Now use the given value of b and $a=a_1$ to introduce damping into the discussion. What happens to the long-term behavior of solutions in this case? Determine the value of the damping parameter that separates the underdamped case from the overdamped case.
3. SOFT SPRING WITH NO DAMPING. Consider the soft spring that corresponds to the positive value a_2 of a. Over what range of y-values is this model reasonable? Consider the periods of the periodic solutions that have the initial condition $v(0)=0$. Sketch the graph of the period as a function of the initial condition $y(0)$. Is there a minimum period? A maximum period? Use the phase portrait to help justify your answer.
4. SOFT SPRING WITH DAMPING. Using the given values of b and $a=a_2$, what happens to the long-term behavior of solutions in this case? Determine the value of the damping parameter that separates the underdamped case from the overdamped case.
5. THE PHYSICAL POINT OF VIEW. What's the difference between a hard spring and a soft spring?

YOUR REPORT:
Address each of the five items in the form of short paragraphs. You may illustrate your paragraphs with phase portraits and graphs of solutions. However, your paragraphs should be complete and understandable without the pictures. Make sure you relate the behavior of the solutions to the motion of the associated mass and spring systems.

TABLE 5.1 Parameter values.
(Assume the mass $m=1$.)

Choice	k	b	a_1	a_2
1	0.1	0.15	−0.005	0.005
2	0.2	0.20	−0.008	0.008
3	0.3	0.20	−0.009	0.009
4	0.2	0.20	−0.005	0.005
5	0.1	0.10	−0.005	0.005
6	0.3	0.20	−0.007	0.007
7	0.3	0.15	−0.007	0.007
8	0.1	0.15	−0.004	0.004
9	0.2	0.15	−0.005	0.005