MAT 4310	Final Exam	NAME:____________________
SPRING '13	Form FE	EMAIL:____________________

Following directions closely answering all questions completely.

§I. PROBLEMS. *You must show your work to receive credit.* There are 5 problems at 20 points each.

1. Recall the *central difference formula* (see *C-K* #3, pg 15) is

$$f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}.$$

In Python, modify and run the code given in *First* (see *C-K* pg 10) so that values for the *rounding error* and the *truncation error* are printed. Plot (*you can use Maple or another grapher*) the rounding error, truncation error, and total error (round + truncation) on a graph using a log scale [i.e., $(x,y) = (\log_{10}(h), -\log_{10}(|error|))$.] Describe what your graph indicates about the errors.

2. Calculate the *Romberg array* needed to estimate $\int_0^1 \frac{4\,dx}{1+x^2}$ with $R(4,4)$. What is the error from the exact value?

3. Use *Gaussian elimination with partial pivoting* to solve the system

$$\begin{cases} x_1 + 3x_2 + 2x_3 + x_4 = -2 \\ 4x_1 + 2x_2 + x_3 + 2x_4 = +2 \\ 2x_1 + x_2 + 2x_3 + 3x_4 = +1 \\ x_1 + 2x_2 + 4x_3 + x_4 = -1 \end{cases}$$

4. (a) Convert the initial value problem $\{y'' + y' - 2y = 0,\ y(0) = 0,\ y'(0) = 3\}$ to a system of first-order differential equations.
   (b) Find $y(3)$ to 6 decimal places using a Runge-Kutta 4-5 method (*RK*–45).

5. Use the *simplex method* to solve the linear program

$$\begin{aligned} \text{Maximize} \quad & z = 2x_1 + 4x_2 + 3x_3 \\ \text{subject to} \quad & x_1 + 3x_2 + 2x_3 \le 30, \\ & x_1 + x_2 + x_3 \le 24, \\ & 3x_1 + 5x_2 + 3x_3 \le 60, \\ & x_i \ge 0 \end{aligned}$$

   (a) Show all simplex tableaux.
   (b) Interpret all the *shadow prices* and *reduced costs*.

☐ EC: What two countries GINI Index bracket the index of the US?