Gauss-Kronrod Quadrature in Practice

$GK_{7,15}$ (1989)

A widely used implementation is based on a Gaussian quadrature with 7 nodes. Kronrod adds 8 to total 15 nodes.

$$G_7 = \sum_{k=1}^{7} w_k f(x_k)$$

$$GK_{7,15} = \sum_{k=1}^{15} w_k f(x_k)$$

$$\varepsilon_{7,15} \approx \left| G_7 - GK_{7,15} \right|$$

or, in practice, use[10]

$$\approx \left[200 \left| G_7 - GK_{7,15} \right| \right]^{3/2}$$

$GK_{7,15}$ on $[-1,1]$

Gauss-7 nodes	Weights
0.00000 00000 00000	0.41795 91836 73469
±0.40584 51513 77397	0.38183 00505 05119
±0.74153 11855 99394	0.27970 53914 89277
±0.94910 79123 42759	0.12948 49661 68870

Kronrod-15 nodes		Weights
0.00000 00000 00000	*G*	0.20948 21410 84728
±0.20778 49550 07898	*K*	0.20443 29400 75298
±0.40584 51513 77397	*G*	0.19035 05780 64785
±0.58608 72354 67691	*K*	0.16900 47266 39267
±0.74153 11855 99394	*G*	0.14065 32597 15525
±0.86486 44233 59769	*K*	0.10479 00103 22250
±0.94910 79123 42759	*G*	0.06309 20926 29979
±0.99145 53711 20813	*K*	0.02293 53220 10529

[10] Kahaner, Moler, & Nash, *Numerical Methods and Software*, Prentice-Hall, 1989.