MAT 4310	Test 1	NAME:____________
SPRING '13	Form A	EMAIL:____________

Work quickly and carefully, following directions closely. Answer all questions completely.

§I. TRUE and/or FALSE. Circle your answer. There are 3 questions at 2 points each.

1. TRUE or FALSE: The values $x = 1{,}000$ and $y = 3.141$ have the same number of *significant digits*.

2. TRUE or FALSE: *Lagrange's Remainder* for a Taylor polynomial of degree n centered at c is

$$R_{n+1} = \frac{1}{(n+1)!} f^{(n+1)}(\xi_x)\,(x-c)^{n+1}$$

where ξ_x is some value between x and c.

3. TRUE or FALSE: Let `s` be a Python list. The Python statements `s.reverse()` and `s[-1::-1]` are essentially equivalent.

§II. MULTIPLE CHOICE. Circle your answer. There are 3 question at 5 points each.

1. The best choice form for evaluating $r_- = \dfrac{-b-\sqrt{b^2-4ac}}{2a}$ when $ac \approx 0$ is

(a) $r_- = \dfrac{-b-\sqrt{b^2-4ac}}{2a}$ (b) $r_- = -\dfrac{b}{2a} - \sqrt{\dfrac{b^2}{4a^2} - \dfrac{c}{a}}$ (c) $r_- = \dfrac{4ac}{-b+\sqrt{b^2-4ac}}$

(d) none of the above (e) all of the above

2. Suppose that `s = range(14)`. Then `s[-1:2:-3]` returns

(a) `[13,10,7,4]` (b) `[2,5,8,11]` (c) `[ ]`

(d) none of the above (e) all of the above

3. For an arbitrary differentiable function f:

(a) *Newton's method* must always find a root.
(b) The *secant method* must always find a root.
(c) If *Newton's method* fails, then the *secant method* must also fail.
(d) none of the above are true.
(e) all of (a), (b), and (c) above are true.

§III. PROBLEMS. *You must show your work to receive credit.* There are 5 problems at 15 points each.

1. Describe *Gauss-Kronrod quadrature* and how the method's error is estimated.

2. Describe *Romberg integration*; state how the method is calculated.

3. Write the Heaviside function $H(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases}$ in Python.

4. Convert the Maple procedure below to a Python function.

Maple	Python

```
EulerStep := proc(yp,x,y,h,n)
   # yp: f'(x,y); h:Delta_x; n: num steps
   local x0, x1, y0, y1, i;
   x0 := x;
   y0 := y;
   for i to n do
      x1 := x0 + h;
      y1 := y0 + yp(x0,y0)*h;
      x0 := x1;
      y0 := y1;
      end do;
   return([x0,y0])
   end proc:
```

5. The *Intermediate Value Theorem* shows that $f(x) = x^3 + x - 1$ has a root between $x = 0$ and $x = 1$. Find an inequality for the error ε_{n+1} in terms of ε_n and the constant $c(\delta)$ [give a specific value for $c(\delta)$] for using *Newton's method* to find the root of the function.