MAT 4310	Test 2	NAME:__________________
SPRING '13	Form T	EMAIL:__________________

Work quickly and carefully, following directions closely. Answer all questions completely.

§I. TRUE and/or FALSE. Circle your answer. There are 3 questions at 2 points each.

1. TRUE or FALSE: The Taylor method of order 4 is equivalent to a Runge-Kutta fourth order method.

2. TRUE or FALSE: Gaussian elimination is a method of finding derivatives of high-order polynomials.

3. TRUE or FALSE: Partial pivoting is used to optimize a matrix method of solving systems of linear equations.

§II. MULTIPLE CHOICE. Circle your answer. There are 3 question at 5 points each.

1. Set $z_0 = f(x_0, y_0)$. The second order Taylor series for a function of two variables $f(x,y)$ is
   (a) $T(x,y) = z_0 + f_{xy}(x_0,y_0)\cdot(x-x_0)\cdot(y-y_0)$
   (b) $T(x,y) = z_0 + [f_x(x_0,y_0)+f_y(x_0,y_0)]\cdot(x-x_0)(y-y_0) + [f_{xx}(x_0,y_0)+f_{yy}(x_0,y_0)]\cdot(x-x_0)^2(y-y_0)^2$
   (c) $T(x,y) = z_0 + f_x(x_0,y_0)\cdot(x-x_0) + f_x(x_0,y_0)\cdot(y-y_0) + f_{xx}(x_0,y_0)\cdot(x-x_0)^2 + f_{xy}(x_0,y_0)\cdot(x-x_0)(y-y_0)$ $+ f_{yy}(x_0,y_0)\cdot(y-y_0)^2$
   (d) none of the above.

2. *Euler's method* will always converge to a unique solution for any differential equation:
   (a) given a small enough stepsize.
   (b) given a differential equation that has a closed-form-formula as its solution.
   (c) given the parameters are all rational numbers.
   (d) None of the above.
   (e) All of the above are true.

3. An adaptive solver for a differential equation:
   (a) must always find a root.
   (b) changes the differential equation from a high order to a system of first order equations.
   (c) changes the parameters/variables from rational numbers to floating point.
   (d) changes the stepsize in a numeric routine to increase accuracy.
   (e) All of the above are true.

§III. PROBLEMS. *You must show your work to receive credit.* There are 3 problems at 20 points each.

1. Using Gaussian elimination with partial pivoting, showing all steps, to solve the system

$$\begin{aligned} x_1 + \tfrac{1}{2}x_2 + \tfrac{1}{3}x_3 + \tfrac{1}{4}x_4 &= 1 \\ \tfrac{1}{2}x_1 + \tfrac{1}{3}x_2 + \tfrac{1}{4}x_3 + \tfrac{1}{5}x_4 &= 0 \\ \tfrac{1}{3}x_1 + \tfrac{1}{4}x_2 + \tfrac{1}{5}x_3 + \tfrac{1}{6}x_4 &= 0 \\ \tfrac{1}{4}x_1 + \tfrac{1}{5}x_2 + \tfrac{1}{6}x_3 + \tfrac{1}{7}x_4 &= 0 \end{aligned}$$

2. A model for fluid draining from a box has the differential equation (see the *J. of Engineering Math.* 79:1, pp 91–99):

$$z''' = \frac{3}{z^2} - \frac{2}{z^3}$$

   (a) Explain whether or not there must be a unique solution in a neighborhood of
      i. $t = 0$.
      ii. $t = 1$.
      iii. $t = 2$.

   (b) Convert this DE to a system of first order differential equations.

3. Define the *IVP*

$$\begin{cases} y' = 5 \cdot y \cdot (1 - y) + 5 \cdot U(5 - x) \\ \quad y(0) = 0 \end{cases}$$

   where the Heaviside function U is given by $U(z) = \begin{cases} 1 & z \geq 0 \\ 0 & z < 0 \end{cases}$

   (a) Use a second order Taylor method to determine $y(15)$ with stepsizes 2^{-k} for $k = 2, 3, 4$. Plot your results.

   (b) Use an RK45 method to determine $y(15)$ with stepsizes 2^{-k} for $k = 2, 3, 4$ giving error estimates. Plot your results.

   (c) Compare your answers from (a) and (b).

EC: What do *coffee*, *Mocha Dick*, and an *English teacher* have in common?