Properties of Vector Spaces

Theorem 1 *Let X be a vector space over the field F. Let $x, y, z \in X$ and $\alpha, \beta \in F$. Then*

1. *if $\alpha x = \alpha y$ and $\alpha \neq 0$, then $x = y$;*
2. *if $\alpha x = \beta x$ and $x \neq 0$, then $\alpha = \beta$;*
3. *if $x + y = x + z$, then $y = z$;*
4. *$\alpha \cdot 0 = 0$;*
5. *$\alpha(x - y) = \alpha x - \alpha y$ where $-y \stackrel{\Delta}{=} (-1) \cdot y$;*
6. *$(\alpha - \beta)x = \alpha x - \beta x$;*
7. *$x + y = 0$ implies that $x = -y$.*

More Examples of Vector Spaces

Sequence Vector Spaces

- $\mathbb{R}^\infty$ and $\mathbb{C}^\infty$
- Finitely non-zero real (or complex) sequences
- Null real (or complex) sequences
- Bounded real (or complex) sequences
- Convergent real (or complex) sequences

Function Vector Spaces

- $\mathbb{P} = \{$polynomials with real (or complex) coefficients$\}$
- $C([a,b]) = \{f \mid f : [a,b] \to \mathbb{R}$ is continuous$\}$ over $\mathbb{R}$
- $L_1([a,b]) = \{f \mid \int_a^b |f(t)|\, dt < \infty\}$ over $\mathbb{R}$