

Project Solution

The **Group-Project** solution is much easier when looking at the spaces from a different “dimension.”

The Setup. Define $\mathcal{D} : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ by

$$\mathcal{D}([x_1, x_2, x_3, x_4]) = [x_2, 2x_3, 3x_4, 0]$$

A Solution. Consider $\mathcal{T} : \mathbb{P}^3 \rightarrow \mathbb{P}^3$ with $\mathcal{T}(p) = p'$. ($\mathbb{P}^3 \cong \mathbb{R}^4$)

- $\mathfrak{R}(\mathcal{T}) = \{\text{polynomials of degree } 2\} \cong \mathbb{R}^3$
- $\mathfrak{N}(\mathcal{T}) = \{\text{constant polynomials}\} \cong \mathbb{R}^1$
- $4 = 3 + 1 \Rightarrow \mathbb{R}^4 \cong \mathbb{P}^3 = \{p \in \mathbb{P}^3 \mid p(0) = 0\} \oplus \mathfrak{N}(\mathcal{T})$