29. Similarity & Equivalence

Ab hinc: X and Y are vector spaces over F with $\dim X = n$ and $\dim(Y) = m$.

Theorem 61 *Let $\mathcal{B}_X = \{e_1, \ldots, e_n\}$ be a basis for X and let $\mathbf{P} = [p_{ij}]$ be an $n \times n$ matrix. Set $e'_k = \sum_j p_{jk} e_j$. Then $\mathcal{B}'_X = \{e'_1, \ldots, e'_n\}$ is a basis for X iff $\mathbf{P}$ is nonsingular.*

Pf. *Calculation based on the linear independence of $\mathcal{B}_X$.*

Definition 43 *Let $\mathbf{P}$ be the matrix of Thm 61, then $\mathbf{P}$ is* the matrix of $\mathcal{B}'_X$ w.r.t $\mathcal{B}_X$.

Theorem 62 *$\mathbf{P}$ is the matrix of $\mathcal{B}'_X$ w.r.t $\mathcal{B}_X$ iff $\mathbf{P}^{-1}$ is the matrix of $\mathcal{B}_X$* w.r.t $\mathcal{B}'_X$.

Pf. *Exercise*.

Similarity of Matrices

Theorem 63 *Let $\mathbf{P}$ be the matrix of $\mathcal{B}'_X$ w.r.t $\mathcal{B}_X$ and $\mathbf{Q}$ be the matrix of $\mathcal{B}''_X$ w.r.t $\mathcal{B}'_X$. Then $\mathbf{PQ}$ is the matrix of $\mathcal{B}''_X$ w.r.t $\mathcal{B}_X$.*

Pf. *Exercise*.

Theorem 64 *Let $\mathbf{P}$ be the matrix of $\mathcal{B}'_X$ w.r.t $\mathcal{B}_X$ and let $x \in X$ be $\mathbf{x}$ in $\mathcal{B}_X$ coordinates. Then $\mathbf{P}\mathbf{x}' = \mathbf{x}$ gives x in $\mathcal{B}'_X$ coordinates.*

Pf. *Exercise*.

$$\begin{array}{ccc} \mathbf{x} = \mathbf{P}\mathbf{x}' & \xrightarrow{\mathbf{A}} & \mathbf{y} = \mathbf{A}\mathbf{x} \\ \uparrow \mathbf{P} & & \downarrow \mathbf{Q} \\ \mathbf{x}' & \xrightarrow{\mathbf{A}'} & \mathbf{y}' = \mathbf{Q}\mathbf{y} \\ & & = \mathbf{QAP}\mathbf{x}' \end{array}$$

(Go to TOC)