34. "Eigen-Basis" Examples

Example 36

1. *Let* $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \in L(\mathbb{R}^3, \mathbb{R}^3)$. *Then* $p(\lambda) = \lambda^3 - 2\lambda^2 + \lambda$ *(and* $m(\lambda) = \lambda^2 - \lambda$*) which indicates that* A *has eigenvalues:* $0, 1, 1$. *The corresponding eigenvectors come from* $\mathfrak{N}_\lambda = \mathfrak{N}(A - \lambda I)$. *So* $\mathfrak{N}_0 = \left\langle \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\rangle$ *and* $\mathfrak{N}_1 = \left\langle \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\rangle$. *(Found by solving* $\begin{bmatrix} 1-\lambda & 0 & 0 \\ 1 & -\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$ *with* $\lambda = 0$ *and* 1, *respectively.)*
   - (a) *Define* P *and find* P^{-1}.
   - (b) *Calculate the diagonal matrix* $P^{-1} A P$ without *using matrix multiplication.*

“Eigen-Basis” Examples, II

Example 37

1. *Let* $B = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix} \in L(\mathbb{R}^3, \mathbb{R}^3)$. *Then* $p(\lambda) = \lambda^3 - 3\lambda^2$ *(and* $m(\lambda) = \lambda^3 - 3\lambda^2$*) which indicates that* B *has eigenvalues:* $0, 0,$ *and* 3. *The corresponding eigenvectors come from* $\mathfrak{N}_\lambda = \mathfrak{N}(B - \lambda I)$. *So* $\mathfrak{N}_0 = \langle \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \rangle$ *and* $\mathfrak{N}_3 = \langle \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \rangle$. *(Found by solving* $\begin{bmatrix} 1-\lambda & 1 & 2 \\ 1 & 1-\lambda & 2 \\ 1 & 0 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$ *with* $\lambda = 0$, *and* 3, *respectively.)*
   (a) *Explain why* P *(and so* P^{-1}*) doesn't exist.*
   (b) *Can* B *be diagonalized? Why or why not?*

(Solution.)

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