Complex) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$$\cancel{A - \lambda I = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0-\lambda 1 & 1 \\ -1 & 0-\lambda 1 \end{bmatrix}}$$

$\det(A - \lambda I) = \lambda^2 + 1$ – NO REAL ROOTS COMPLEX

DUE TO FUND Th^{m} of Algebra "Every poly complex coeff can be factored"

THUS complex eigenvector and complex eigen values

Gauss father

Curves $ax^2 + bxy + cy^2 = d$ (Quadratic Curves) ← planets comets

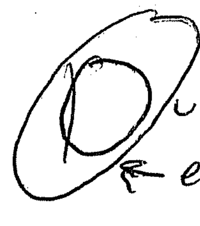
Matrix form $X^t M X = d$ where $X = [x, y]^t$

and $M = \begin{bmatrix} a & b/2 \\ b/2 & c \end{bmatrix}$

Very Yucky but very simple if we use coordinates defined by the eigenvectors of the matrix.

corresponding vectors

$$\lambda_1 (x') + \lambda_2 (y')^2 = d$$



unit circle

← eignvectors

where λ are the eigenvalues of M (orthonormal basis)

Definition) Let A be an $n \times n$ matrix. A non-zero vector X such that $AX = \lambda X$ for some scaler λ is called <u>eigenvector</u> for A. The scaler λ is <u>called</u> eigen values

How to find them)

1) Find the roots of the polynomial $p(\lambda) = \det(A - \lambda I)$. This characteristic polynomial gives you the eignvalues.

2) For each eignvalue λ, we find all solutions to ~~(A−λ...)~~ $(A - \lambda I)X = 0$. Now we have eigenvectors

NOW WE HAVE A BASIS FOR THE SOLUTION SPACE

Ex) Let $A = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 5 \end{bmatrix}$ YES! △ we can skip the I.

(I) so $\det(A - \lambda I) = (3-\lambda)(3-\lambda)(5-\lambda)$

$= (3-\lambda)^2(5-\lambda)$

thus $\lambda = 3$ $\lambda = 5$

(II) $(A - 3I)X = 0$

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

solution $X = x_1[1, 0, 0]^t$

One basis element $X = [1, 0, 0]^t$

Similarly $(A - 5I)X = 0$ $x_3[\frac{3}{4}, \frac{1}{2}, 1]^t$ so $Y = [3, 2, 4]^t$

NOTE: ONLY TWO INDPEDENT VECTORS

TWO IND VECTORS DONT SPAN $\mathbb{R}^3$

* $n \times n$ matrix that does not have n ind ~~vecto~~ eigenvectors is called deficient,

why? multiplicity root,

thm If λ is an eigenvalue of multiplicity n then the corresponding eigenspace can be at most n dimensional LTTRAAE

SO WHAT & HOW CAN I APPLY

COMPUTE FORMULAS! For EXAMPLE TRENDS that continue for a long time

let $A^m B$ where B was a given column vector and A is an $n \times n$ matrix. If we can find scalers c_i s.t. $B = c_1 X + \dots + c_i X_i$ where X_i are eigenvectors, then

$$A^m B = c_1 \lambda_1^m X_1 + \cdots + c_i \lambda_i^m X_i$$

where the λ_i are the eigenvalues corresponding to the X.

CATCH OF THE DAY

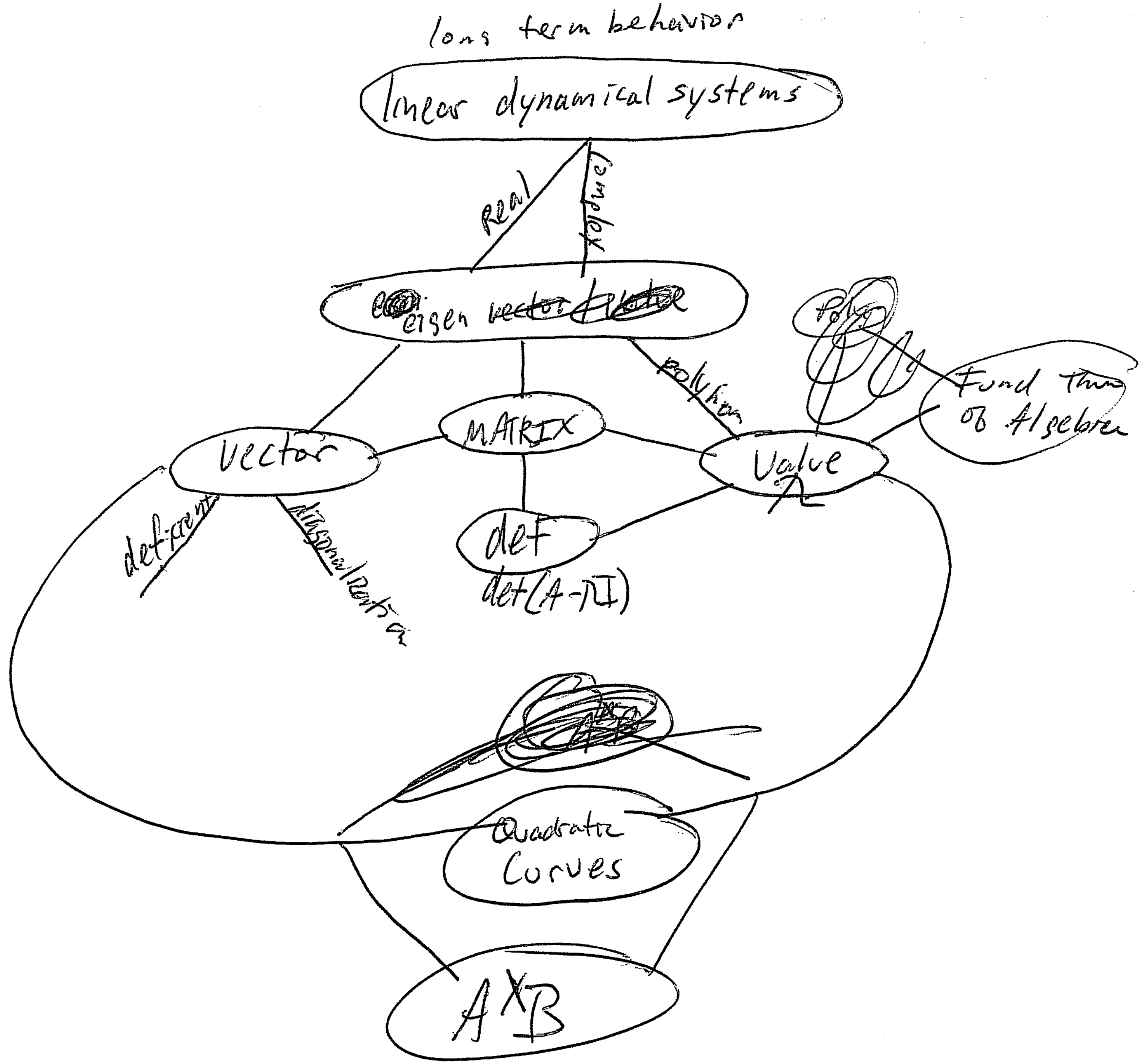
this can be done for all B in $\mathbb{R}^n$ if the X_i form a basis for $\mathbb{R}^n$, the A is called diagonalizable.

Back to our example Let $B = [1,1,1]^t$ ~~Attempt~~ Compute $A^{100}B$

can we express B as a linear combination of the eigenvectors?

$$[1,1,1]^t = a[1,0,0] + b[3,2,4] \quad \text{NOPE}$$

So we cannot compute $A^{100}B$ NOTE: IF WE could NO MATRIX MULTIP


long term behavior
linear dynamical systems
real
complex
eigen
polynom
Vector
MATRIX
Value
λ
Fund Thm
of Algebra
def
$det(A-\lambda I)$
diagonal/rotation
deficient
Quadratic
Curves
$A \times B$