MAT 5230 Homework 1 Due: Aug 29

1. Let  $M_2$  be the set of  $2 \times 2$  matrices with real entries using the usual matrix multiplication and addition.

- (a) Show that  $M_2$  is a ring.
- (b) Show that  $M_2$  is not a field.
- 2. Let  $D_2$  be the set of  $2 \times 2$  diagonal matrices with real entries using the usual matrix multiplication and addition.
  - (a) Show that  $D_2$  is closed under multiplication.
  - (b) Show that multiplication is commutative in  $D_2$ .
  - (c) Show that  $D_2$  is a ring.
  - (d) Show that  $D_2$  is not a field. (Hint: What's a zero-divisor?)
- 3. Let  $T_2$  be the set of  $2 \times 2$  diagonal matrices with real entries having nonzero determinants together with the zero matrix using the usual matrix multiplication and addition.
  - (a) Is  $T_2$  a ring?
  - (b) Is  $T_2$  a field?