1. Let $M_{2}$ be the set of $2 \times 2$ matrices with real entries using the usual matrix multiplication and addition.
(a) Show that $M_{2}$ is a ring.
(b) Show that $M_{2}$ is not a field.
2. Let $D_{2}$ be the set of $2 \times 2$ diagonal matrices with real entries using the usual matrix multiplication and addition.
(a) Show that $D_{2}$ is closed under multiplication.
(b) Show that multiplication is commutative in $D_{2}$.
(c) Show that $D_{2}$ is a ring.
(d) Show that $D_{2}$ is not a field. (Hint: What's a zero-divisor?)
3. Let $T_{2}$ be the set of $2 \times 2$ diagonal matrices with real entries having nonzero determinants together with the zero matrix using the usual matrix multiplication and addition.
(a) Is $T_{2}$ a ring?
(b) Is $T_{2}$ a field?
