

**MAT 5230 Homework****Due: Oct 7**

1. Finish the proof of Theorem 3.4.77 (pg 108).
2. Prove Theorem 3.4.78 (pg 108).
3. Define  $X$  to be the vector space  $\mathbb{R}^3$  and  $Y$  to be the vector space

$$\mathbb{P}_2^2 = \{p \mid p(x) \text{ is a polynomial of degree 2 with coefficients in } \mathbb{Z}_2\}.$$

Then  $\mathcal{B}_X = \{e_1 = [1, 0, 0], e_2 = [0, 1, 0], e_3 = [0, 0, 1]\}$  forms the *standard basis* for  $X$ ; also  $\mathcal{B}_Y = \{1, x, x^2\}$  forms the *standard basis* for  $Y = \mathbb{P}_2^2$ .

- (a) Find  $\dim(X)$ .
  - (b) Find  $\dim(Y)$ .
  - (c) **True or False:**  $X \cong Y$ . That is,  $\mathbb{R}^3 \cong \mathbb{P}_2^2$ . (Explain your reasoning.)
4. **True or False:** For  $n \in \mathbb{Z}^+$ , we have  $\mathbb{C}^n \cong \mathbb{R}^{2n}$ . (Explain your reasoning.)