MAT 5230 **Homework 2** **Due: Sept 7**

1. Let X be a vector space over F with $x, y \in X$ and $\alpha \in F$.
   (a) Prove that $\alpha \cdot 0 = 0$.
   (b) Prove that $\alpha \cdot (x - y) = \alpha \cdot x - \alpha \cdot y$.

2. Let $\mathbb{R}_B^\infty$ be the vector space of bounded real sequences.
   (a) Prove that $\mathbb{R}_B^\infty$ is closed under addition and scalar multiplication.
   (b) Prove that addition is commutative in $\mathbb{R}_B^\infty$.

3. Let $\mathbb{P}_3^*$ be the set of polynomials of degree 3 or less that have constant term equal to 0; i.e., $p(x) = a_3x^3 + a_2x^2 + a_1x + 0$.
   (a) Discuss (informally) whether or not $\mathbb{P}_3^*$ is a vector space.