MAT 5320	Exam 1	NAME:____________________
AUTUMN '05	**Due: Thursday, Oct 27, at 5:00 pm.**	STUDENT ID #:____________________

You must work alone. You may use the text, other books, your notes, etc. and your instructor; you may not consult another person in any way. Answer all questions completely.

§I. TRUE and/or FALSE. There are 5 questions at 2 points each. Circle your answer.

1. TRUE or FALSE. The set of all complex-valued polynomials on the interval $[-1,+1]$ is a subspace of $\mathcal{C}_{\mathbb{C}}[-1,+1]$, the space of continuous complex-valued functions on $[-1,+1]$.
2. TRUE or FALSE. Let $n \in \mathbb{Z}^+$. Every n-dimensional vector space has a basis containing the zero vector 0.
3. TRUE or FALSE. If $A = \{x_1, x_2, \ldots, x_n\} \subset X$ is linearly independent and $x \notin \mathcal{V}(A)$, then it follows that the set $A_x = \{x, x_1, x_2, \ldots, x_n\}$ is linearly independent.
4. TRUE or FALSE. Let X be a finite dimensional vector space. Let $\mathcal{I}$ be the collection of linearly independent subsets of X; let $\mathcal{S}$ be the collection of spanning subsets of X. Then $\mathcal{I} \cap \mathcal{S} \neq \emptyset$.
5. TRUE or FALSE. The set of linear transformations $L(X,X)$ is a subspace of the space of linear functionals X^f.

§II. PROBLEMS. There are 6 problems at 20 points each.

1. Set $Y = \{[1,0,1,0], [1,1,1,0], [3,2,1,0], [1,2,3,0]\} \subset \mathbb{R}^4$.
   (a) Demonstrate that Y is linearly dependent.
   (b) Prove that $\mathcal{V}(Y) \cong \mathbb{R}^3$.
2. Determine which of the following are linear transformations.
   (a) The mapping $R : \mathbb{R}^4 \to \mathbb{R}^2$ given by $R([x_1, x_2, x_3, x_4]) = [x_1 + x_2, x_3 + x_4]$.
   (b) The mapping $S : \mathbb{R}^3 \to \mathbb{R}^3$ given by $S([x_1, x_2, x_3]) = [x_2, x_3, x_1]$.
   (c) The mapping $T : \mathbb{R}^2 \to \mathbb{R}^1$ given by $T([x,y]) = x\ \tan(y)$.
3. Suppose that the linear transformation $\phi : \mathbb{R}^3 \to \mathbb{R}^3$ is given by $\phi([x,y,z]) = [2x, x - z, 2z]$.
   (a) Find $\phi([1,0,0])$, $\phi([0,1,0])$, and $\phi([0,0,1])$.
   (b) Find the range space $\mathfrak{R}(\phi)$ and the rank $\rho(\phi)$.
   (c) Find the null space $\mathfrak{N}(\phi)$ and the nullity $\nu(\phi)$.
   (d) Determine $\mathfrak{R}(\phi) \cap \mathfrak{N}(\phi)$.
4. Prove:

   Theorem. *Let F be a field and $n \in \mathbb{Z}^+$. Then F^n is a vector space.*
5. Prove:

   Theorem. *Let $T \in L(X,Y)$. Then T is injective if and only if $\mathfrak{N}(T) = \{0\}$.*
6. Prove:

   Theorem. *Let X be a vector space with a subspace X_i for each $i \in \mathcal{I}$ for some index set $\mathcal{I}$. Then the intersection $\mathcal{X} = \bigcap_{i \in \mathcal{I}} X_i$ is a subspace of X.*

EC: Give the author, the play, the act, and the scene for the audio clip on the web site.