MAT 5320	Final Exam	NAME:____________________
AUTUMN '05	**Due: Friday, Dec 2, at 12:00 pm.**	STUDENT ID #:____________________

You must work alone. You may use the text, other books, your notes, etc. and your instructor; you may not consult another person in any way. Answer all questions completely.

§I. TRUE and/or FALSE. There are 2 questions at 2 points each. Circle your answer.

1. TRUE or FALSE. For any finite $n \in \mathbb{Z}^+$, there is only one real vector space of dimension n.

2. TRUE or FALSE. Each $m \times n$ matrix with elements in $\mathbb{Z}_5$ defines a unique linear transformation.

§II. PROBLEMS. There are 4 problems at 20 points each.

1. Let $A \in L(\mathbb{R}^4, \mathbb{R}^4)$ be given by $A = \begin{bmatrix} 0 & -1 & 0 & 2 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix}$. Find
   (a) $p(\lambda)$
   (b) $m(\lambda)$
   (c) the eigenvalues and their algebraic & geometric multiplicities
   (d) the transition matrix P
   (e) $JCF(A)$

2. Determine all possible Jordan Canonical Forms for a 5×5 matrix A
   (a) whose characteristic polynomial is $p(\lambda) = (2 - \lambda)^3(5 - \lambda)^2$.
   (b) whose minimal polynomial is $m(\lambda) = (4 - \lambda)^2$.

3. Prove:

   Theorem. *Define the $k \times k$ matrix*

$$N_k = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & & \dots & 0 & 1 \\ 0 & & \dots & 0 & 0 \end{bmatrix}.$$

   Then N_k is nilpotent of index k.

4. Prove:

   Theorem. *Let $A \in \mathfrak{M}_{n \times n}$. A is nonsingular iff the characteristic polynomial has nonzero constant term; i.e., $p(0) \neq 0$.*

EC: Who said: "You can observe a lot just by watchin'."