Test 1 Minimal List of Essential Knowledge

Important Definitions

- Field
- Vector Space
- Isomorphism
- Subspace
- Direct Sum
- Linear Combination
- Span
- Linear Dependence/Independence
- Basis
- Dimension
- Linear Transformation
- Null Space
- Range Space
- Nullity
- Rank
- Injective, Surjective, Bijective
- Singular, Nonsingular
- Linear Functional

Important Theorems (paraphrased)

- For every prime p and $n \in \mathbb{Z}$, there exists a unique field with p^n elements.
- The intersection of subspaces is a subspace.
- The span of a set is a subspace.
- The maximum size of a linearly independent set equals the dimension of the space.
- Every basis of a finite dimensional space X contains $\dim(X)$ vectors.
- For $T \in L(X, Y)$, we have

$$\dim(\mathfrak{R}(T)) + \dim(\mathfrak{N}(T) = \dim(X);$$

$$\rho + \nu = n$$

- For $T \in L(X, Y)$, we have T^{-1} exists iff $\mathfrak{N}(T) = \{0\}$ iff $\rho(T) = \dim(X)$.
- $L(X, X)$ is an associative algebra.
- Every finite dimensional vector space X over F is isomorphic to $F^{\dim(X)}$.
- X^f is a vector space (called the conjugate space of X).

Theorems (paraphrased) for Proving

- $\mathbb{R}^n$ (or $\mathbb{C}^n$) is a vector space.
- $\mathcal{C}[a, b]$ is a vector space.
- A subspace of a vector space is a vector space.
- The intersection of a collection of subspaces is a subspace.
- If Y is linearly independent, then Y is a basis for $V(Y)$, the span of Y.
- If $\{e_i\}$ is a basis and $x \in X$, then there are unique scalars α_i such that $x = \sum_n \alpha_i e_i$.
- Let X be a finite dimensional vector space. Then $X \cong F^{\dim(X)}$.
- Let $T \in L(X, Y)$, then T is injective iff $\mathfrak{N}(T) = \{0\}$.
- $\int_0^1 f(t)dt$ is a linear functional on $\mathcal{C}[0, 1]$.
- X^f is a vector space.

Plus something you've never seen.