MAT 5610	Su '03	Name: ____________
Exam 2		Student ID #: ____________

Problems

1. Calculate the integral: $\displaystyle\int_0^1 \frac{e^{\sqrt{x}}}{2}\,dx$.

2. Find the derivative, $F'(x)$, when $F(x) = \displaystyle\int_{-x}^{+x} e^{-t^2}\,dt$.

3. Evaluate the limit: $\displaystyle\lim_{n\to\infty} n \sum_{k=0}^{\infty} \frac{2^2}{n^2+k^2}$.

4. Let $\chi(x)$ be the characteristic function of the rationals; i.e., $\chi(x) = \left\{ \begin{array}{cc} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{otherwise} \end{array} \right\}$.
   Explain why χ is not integrable on $[0,1]$.

5. Show that $\Gamma(x+1) = x\,\Gamma(x)$ for $x > 0$.

Proofs

1. **Theorem 1** *Let $f : [a,b] \to \mathbb{R}$ be bounded. Then f is Riemann integrable on $[a,b]$ if and only if for any $\epsilon > 0$ there is a partition P of $[a,b]$ such that*

$$U(P,f) - L(P,f) < \epsilon.$$

2. **Theorem 2 (The Fundamental Theorem of Calculus)** *Let f be an integrable, real-valued function on $[a,b]$ and let F be an antiderivative of f. Then*

$$\int_a^b f(t)\,dt = F(b) - F(a).$$

3. **Definition 1** *We call $f : [a,b] \to \mathbb{R}$ a* ***step function*** *if there exists a partition, P, of $[a,b]$ so that f is contant on each interval of P.*

   Theorem 3 *Let f be a step function of $[a,b]$. Then f is Riemann integrable on $[a,b]$ and*

$$\int_a^b f(\omega)\,d\omega = \sum_{k=1}^{n} f(x_k - 0)\Delta x_k$$

   where $f(x_k - 0) = \displaystyle\lim_{x \to {x_k}^-} f(x)$.