Definitions

Define the terms of the series as $a(n)$

$$a := n \rightarrow (-1)^{\frac{n+1}{2}} \cdot \frac{1-(-1)^n}{2}$$

$$n \rightarrow (-1)^{\frac{1}{2}n+\frac{1}{2}} \left(\frac{1}{2} - \frac{1}{2}(-1)^n\right) \tag{1.1}$$

Define the series $s(n)$ as the sum of the terms

$$s := n \rightarrow \sum_{i=1}^{n} a(i)$$

$$n \rightarrow \sum_{i=1}^{n} a(i) \tag{1.2}$$

Define the Cesàro sums $\sigma(n)$ as the average of the $s(n)$s

$$\sigma := n \rightarrow \frac{1}{n} \sum_{j=1}^{n} s(j)$$

$$n \rightarrow \frac{\sum_{j=1}^{n} s(j)}{n} \tag{1.3}$$

(Note the different variables for sum indexes to prevent evaluation problems later.)

List The Values

Use a *seq* to list several values of $a(n)$, $s(n)$, and $\sigma(n)$

$N := 15:$

$seq(a(n1), n1 = 1..N)$

$$-1, 0, 1, 0, -1, 0, 1, 0, -1, 0, 1, 0, -1, 0, 1 \tag{2.1}$$

$seq(s(n2), n2 = 1..N)$

$$-1, -1, 0, 0, -1, -1, 0, 0, -1, -1, 0, 0, -1, -1, 0 \tag{2.2}$$

$seq(\sigma(n3), n3 = 1..N)$

$$-1, -1, -\frac{2}{3}, -\frac{1}{2}, -\frac{3}{5}, -\frac{2}{3}, -\frac{4}{7}, -\frac{1}{2}, -\frac{5}{9}, -\frac{3}{5}, -\frac{6}{11}, -\frac{1}{2}, -\frac{7}{13}, -\frac{4}{7}, -\frac{8}{15} \tag{2.3}$$

(Note the different index variables to prevent evaluation problems.)

Or get fancy ...

$interface(rtablesize = 20):$

$evalf[4]\left(\left\langle \left\langle \langle n\rangle \mid \langle a_n \rangle \mid \langle s_n \rangle \mid \langle \sigma_n \rangle \right\rangle ; \langle Vector(N, i \rightarrow i) \mid Vector(N, a) \mid Vector(\mathrm{N}, s) \mid Vector(\mathrm{N}, \sigma) \rangle \right\rangle\right)$

$$\begin{bmatrix} n & a_n & s_n & \sigma_n \\ 1. & -1. & -1. & -1. \\ 2. & 0. & -1. & -1. \\ 3. & 1. & 0. & -0.6667 \\ 4. & 0. & 0. & -0.5000 \\ 5. & -1. & -1. & -0.6000 \\ 6. & 0. & -1. & -0.6667 \\ 7. & 1. & 0. & -0.5714 \\ 8. & 0. & 0. & -0.5000 \\ 9. & -1. & -1. & -0.5556 \\ 10. & 0. & -1. & -0.6000 \\ 11. & 1. & 0. & -0.5455 \\ 12. & 0. & 0. & -0.5000 \\ 13. & -1. & -1. & -0.5385 \\ 14. & 0. & -1. & -0.5714 \\ 15. & 1. & 0. & -0.5333 \end{bmatrix} \tag{2.4}$$

Limits

$\lim_{n \to \infty} a(n)$

$$-\frac{1}{2} - \frac{3}{2}\,\mathrm{I} .. \frac{1}{2} + \frac{3}{2}\,\mathrm{I} \tag{3.1}$$

which means a range, therefore Does Not Exist

$\lim_{n \to \infty} s(n)$

$$-1 - \frac{3}{2}\,\mathrm{I} .. \frac{3}{2}\,\mathrm{I} \tag{3.2}$$

The same as above

$\lim_{n \to \infty} \sigma(n)$

$$-\frac{1}{2} \tag{3.3}$$

Aha! Cesàro-convergence to $-\frac{1}{2}$.