Types of Convergence

Let $\{f_n\}$ be a sequence of functions defined on a common measurable domain $E \subseteq \mathbb{R}$.

Definition 1 (Pointwise)**.** *f_n converges to f* pointwise *on E if and only if . . .*

Definition 2 (Uniform)**.** *f_n converges to f* uniformly *on E if and only if . . .*

Definition 3 (Almost Everywhere)**.** *f_n converges to f* almost everywhere *("a.e.") on E if and only if $f_n(x) \to f(x)$ except on a set of measure zero.*

Definition 4 (Almost Uniformly)**.** *f_n converges to f* almost uniformly *on E if and only if for every $\varepsilon > 0$ there is a set E_ε of measure less than ε such that $f_n \to f$ uniformly on E_ε^c, the complement of E_ε.*

Definition 5 (In Mean)**.** *f_n converges to f* in mean *on E if and only if*

$$\lim_{n\to\infty} \int_E |f_n - f| \, d\mu = 0$$

Definition 6 (In L^p)**.** *f_n converges to f* in L^p *on E if and only if*

$$\lim_{n\to\infty} \left[\int_E |f_n - f|^p \, d\mu \right]^{1/p} = 0$$

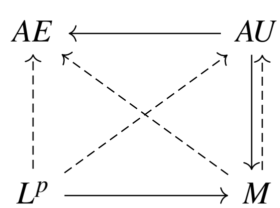
Definition 7 (In Measure)**.** *f_n converges to f* in measure *on E if and only if for every $\varepsilon > 0$*

$$\lim_{n\to\infty} \mu\big(\{x \in E : |f_n(x) - f(x)| > \varepsilon\}\big) = 0$$

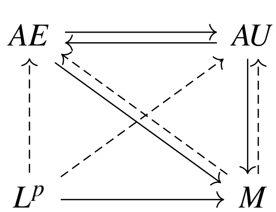
Convergence Diagrams

A solid arrow indicates the source mode implies the target mode. A dashed arrow means that there exists a subsequence that converges in the target mode. Note: AE = almost everywhere, AU = almost uniform, L^p = in L^p mean, M = in measure.

General Measure Spaces



Finite Measure Spaces ($\mu(E) < \infty$)



Dominated Convergence ($\exists g$, integrable, s.t. $|f_n| < g$)