MAT 5620	Homework I	NAME:____________________
SPRING '14		EMAIL ID:____________________

§I. There are 2 proofs at 5 points each.

Proposition 1. *Suppose $\vec{r}(t)$ is integrable on $[a,b]$. Then $\|\vec{r}(t)\|$ is integrable on $[a,b]$ and*

$$\left\| \int_a^b \vec{r}(t)\,dt \right\| \leq \int_a^b \|\vec{r}(t)\|\,dt.$$

Let $\vec{r}(t) = \langle f(t), g(t) \rangle$ for $t \in [a,b]$. Define the *curvature* κ of $\vec{r}$ at t to be

$$\kappa = \left| \frac{d\phi}{ds} \right|$$

where $\phi = \arctan(dy/dx)$ *(the angle between the tangent vector and the x-axis)* and $ds = \sqrt{dx^2 + dy^2}$. Use

$$\left| \frac{d\phi}{ds} \right| = \left| \frac{d\phi/dt}{ds/dt} \right|$$

to prove (remembering that for parametric functions in $\mathbb{R}^2$, $dy/dx = (dy/dt)/(dx/dt)$)

Proposition 2. *For $\vec{r}(t) = \langle f(t), g(t) \rangle$ twice differentiable for $t \in [a,b]$,*

$$\kappa = \frac{\left| \frac{dx}{dt}\frac{d^2y}{dt^2} - \frac{d^2x}{dt^2}\frac{dy}{dt} \right|}{\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 \right]^{3/2}}$$

□