Fundamental Theorem of Calculus

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

Fundamental Theorem of Line Integrals

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_C \nabla f \cdot d\mathbf{r} = f\big(x(b), y(b)\big) - f\big(x(a), y(a)\big)$$

Green's Theorem

$$\int_{\partial R} M\,dx + N\,dy = \iint_R \left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right] dA$$

Alternate Forms

$$\oint_C M\,dx + N\,dy = \oint_C \mathbf{F}\cdot\mathbf{T}\,ds \qquad \textit{Circulation Integral}$$

$$\oint_C M\,dx - N\,dy = \oint_C \mathbf{F}\cdot\mathbf{N}\,ds \qquad \textit{Flux Integral}$$

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \iint_R \operatorname{curl}(\mathbf{F})\cdot\mathbf{k}\,dA \qquad \textit{Curl Integral}$$

$$\int_C \mathbf{F}\cdot\mathbf{N}\,ds = \iint_R \operatorname{div}(\mathbf{F})\cdot\mathbf{k}\,dA \qquad \textit{Divergence Integral}$$

where for $F = M\,\mathbf{i} + N\,\mathbf{j}$

$$\operatorname{curl}(\mathbf{F}) = \tfrac{\partial}{\partial x}N - \tfrac{\partial}{\partial y}M$$

$$\operatorname{div}(\mathbf{F}) = \tfrac{\partial}{\partial x}M + \tfrac{\partial}{\partial y}N = \nabla\cdot\mathbf{F}$$

Divergence Theorem (Also called Gauss's theorem or Ostrogradski's theorem.)

$$\iint_S \mathbf{F}\cdot\mathbf{N}\,dS = \iiint_Q \operatorname{div}(\mathbf{F})\,dV$$

Stoke's Theorem

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \iint_S \operatorname{curl}(\mathbf{F})\cdot\mathbf{N}\,dS$$

Differential Forms Version

THEOREM. Suppose that R is a compact, oriented k-dimensional manifold with boundary ∂R, and let α be a smooth $(k-1)$-form on R. Then

$$\int_R \mathbf{d}\alpha = \int_{\partial R} \alpha$$