

1. Prove:

**Proposition 1.** *Suppose that all of the partial derivatives of  $f$  exist in a neighborhood of  $\vec{p}$  and are continuous at  $\vec{p}$ . Then,  $f$  is differentiable at  $\vec{p}$ .*

2. Suppose that  $f(x, y) = x \cos(\pi y) + y^2 e^x$ . What are the values of the total derivative of  $f$ , at the point  $(-1, 1)$ , with respect to the vectors  $(4, -2)$  and  $(-3, 5)$ , i.e., what are  $D_{(-1,1)}f(4, -2)$  and  $D_{(-1,1)}f(-3, 5)$ ?

3. Show that  $f(x, y) = \sqrt{x^2 + y^2}$  is not differentiable at the origin.

4. (#16 p 400)

Let  $r > 0$ , let  $I(r) = \int_{-r}^{+r} e^{-u^2} du$ .

(a) Show that  $I^2(r) = \iint_R e^{-(x^2+y^2)} dx dy$  where  $R = [-r, r] \times [-r, r]$ .

(b) Let  $C_1$  and  $C_2$  be disks circumscribing and inscribing  $R$ . Show that

$$\iint_{C_1} e^{-(x^2+y^2)} dx dy \leq I^2(r) \leq \iint_{C_2} e^{-(x^2+y^2)} dx dy$$

(c) Change the integrals over  $C_1$  and  $C_2$  to polar coordinates.

(d) Show that  $I^2(r) \rightarrow \pi$ , therefore  $I(r) \rightarrow \sqrt{\pi}$ . Aha! Whence  $\int_0^\infty e^{-u^2} du = \frac{1}{2} \sqrt{\pi}$ .

5. State Green's Theorem for the annulus  $A(\vec{c}, r_{inner}, r_{outer})$ .

6. Compute the double integral  $\iint_R f(x, y) dx dy$  where  $R$  is the unit square  $[0, 1] \times [0, 1]$  and where  $f(x, y) = \sin(\pi(x + y)/2)$ .