Integrating Factors

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Theorem (Integrating Factor)

Consider the ODE $M\,dx + N\,dy = 0$.

1. *If* $\frac{1}{N}\cdot\left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right] = h(x)$, *then set* $\mu(x) = e^{\int h(x)\,dx}$.
2. *If* $\frac{1}{M}\cdot\left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right] = k(y)$, *then set* $\mu(x) = e^{\int k(y)\,dy}$.

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2. The ODE $y' = y/x$ has *IFs:* a) x^{-2}, b) y^{-2}, c) $(xy)^{-1}$, d) $1/(x^2 + y^2)$.