MAT 5620	Final Exam	NAME:
FALL '15		EMAIL:

Work quickly and carefully, following directions closely. Answer all questions completely.

1. Prove or disprove: If $\vec{r}\colon [a,b] \to \mathbb{R}^n$ is integrable, then

$$\left\| \int_a^b \vec{r}(t)\,dt \right\| \leq \int_a^b \|\vec{r}(t)\|\,dt$$

2. Prove: If C is a smooth rectifiable curve, and f is continuous everywhere, then

$$\left| \int_C f\,ds \right| \leq \text{length}(C) \cdot \max_{\vec{x}\in C} |f(\vec{x})|$$

3. Let $\mathscr{S}$ be the surface consisting of the portion of the paraboloid $\Omega = x^2 + y^2$ lying below the plane $z = 1$. Suppose $F(x,y) = -2y\,\mathbf{i} + 2x\,\mathbf{j} + e^z\,\mathbf{k}$. Use Stokes' theorem to calculate the flux $\iint_{\mathscr{S}} \nabla \times F \cdot \vec{n}\,dS$.

4. Prove: Let E be a subset of N where $\mu(N) = 0$. Then $E \in \mathfrak{M}$ and $\mu(E) = 0$.

5. Set $f_n(x) = \dfrac{x^n}{1+x^{2n}}$. Apply Lebesgue's *Dominated Convergence Theorem*, if possible, to determine

$$\lim_{n\to\infty} \int_0^\infty f_n\,d\mu$$

showing how the theorem applies or fails to apply.

6. Prove: Let $f : [0,1] \to \mathbb{R}$ be a non-negative measurable function and $\int_{[0,1]} f\,d\mu = 0$. Then $f = 0$ almost everywhere.

7. Fill in the chart (with 'yes' or 'no'):

Sequence	*Pointwise*	*Uniform*	*Almost Everywhere*	*Almost Uniform*	*In Mean*	*In Measure*
$f_n(x) = \chi_{[n,n+1]}(x)$						
$g_n(x) = \frac{1}{n} \cdot \chi_{[0,n]}(x)$						
$h_n(x) = n \cdot \chi_{[1/n,2/n]}(x)$						
$r_n(x) = \chi_{[j/2^k,(j+1)/2^k]}(x)$ $(n = 2^k + j,\ j = 0..2^k - 1)$						