MAT 5620	Midterm	NAME:________________
SPRING '17		EMAIL:________________

Work quickly and carefully, following directions closely. Answer all questions completely.

1. Prove or disprove: If $\vec{r}: [a,b] \to \mathbb{R}^n$ is integrable, then

$$\left\| \int_a^b \vec{r}(t)\,dt \right\| \leq \int_a^b \|\vec{r}(t)\|\,dt$$

2. Prove: If C is a smooth rectifiable curve, and f is continuous everywhere, then

$$\left| \int_C f\,ds \right| \leq \text{length}(C) \cdot \max_{\vec{x} \in C} |f(\vec{x})|$$

3. Let $\mathscr{S}$ be the surface consisting of the portion of the paraboloid $\Omega = x^2 + y^2$ lying below the plane $z = 1$. Suppose $F(x,y) = -2y\,\mathbf{i} + 2x\,\mathbf{j} + e^z\,\mathbf{k}$. Use Stokes' theorem to calculate the flux $\iint_{\mathscr{S}} \nabla \times F \cdot \vec{n}\,dS$.

4. Fill in the chart with 'Yes' or 'No' and justify your answers[1]:

Sequence	*Pointwise*	*Uniform*	*In Mean*
$f_n(x) = \chi_{[n,n+1]}(x)$			
$g_n(x) = \frac{1}{n} \cdot \chi_{[0,n]}(x)$			
$h_n(x) = n \cdot \chi_{[1/n,2/n]}(x)$			
$r_n(x) = \chi_{[j/2^k,(j+1)/2^k]}(x)$ $(n = 2^k + j,\ j = 0..2^k - 1)$			

[1]Recall: 1. $\chi_A(x) = (1$ if $x \in A$ and 0 otherwise$)$. 2. Define $f_n \to f$ in mean iff $\int_{\mathbb{R}} \left| f_n - f \right| dx \to 0$.