MAT 5620	Final Exam	NAME:________________
FALL '07		ASU EMAIL ID #:______________

1. Prove: If $\vec{u}$ and $\vec{v}$ are vectors in $\mathbb{R}^n$, then $\vec{u} \cdot \vec{v} \leq \|\vec{u}\| \, \|\vec{v}\|$.

2. What is the smallest positive value of a for which the cycloid $f(t) = [a(t - \sin(t)), a(1 - \cos(t))]$ fails to have a tangent slope for $t = \pi$.

3. Suppose that the curve C is parametrized by $\gamma(t) = [x(t), y(t)]$ for $t \in \mathbb{R}$. If dx/dt is continuous and nonzero on $[a, b]$, then C can be written as $y = F(x)$ on the interval from $x(a)$ to $x(b)$.

4. Show that the function $f(x, y) = \dfrac{1}{x + y}$ is not uniformly continuous on $D = (0, 1) \times (0, 1)$.

5. Let $f(x, y) = \begin{cases} (x^2 + y^2) \cdot \sin(1/\sqrt{x^2 + y^2}) & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{otherwise} \end{cases}$.

   (a) Determine whether f is differentiable at $(0, 0)$.

   (b) If f_x or f_y exist at $(0, 0)$, is either (or both) continuous at $(0, 0)$?

6. Suppose that $x(t)$ and $y(t)$ are differentiable for all t. Calculate $\dfrac{d^2 z}{dt^2}$ where $z = f(x, y)$ assuming all necessary derivatives exist.

7. Prove: If there is any set $E \in \mathfrak{M}$ that has $m(E) < \infty$, then the measure of the empty set must be 0.

8. Define $D(x) = \begin{cases} \frac{1}{q} & x = \frac{p}{q}, (p, q) = 1 \\ 0 & \text{otherwise} \end{cases}$.

   (a) Show that D is continuous a.e.

   (b) Show that the Riemann integral $\int_0^1 D(x)\, dx$ is equal to the Lebesgue integral $\int_{[0,1]} D$ and that both are 0.

9. Evaluate $\int_0^b x \, d(x - \lfloor x \rfloor)$ where $b > 0$.

10. Prove: Suppose that f is a measurable function and E is a set with $m(E) < \infty$. If $a \leq f(x) \leq b$ a.e. on E, then $a \cdot m(E) \leq \int_E f \leq b \cdot m(E)$.