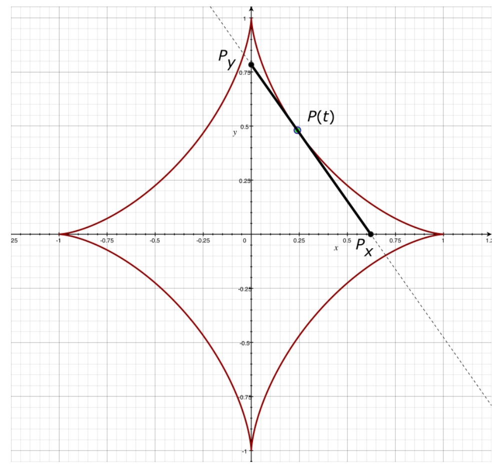
MAT 5620	Exam 1	NAME:________________
FALL '07		ASU EMAIL ID #:________________

Work quickly and carefully, following directions closely. Answer all questions completely.

§I. PROBLEMS.

1. Let C be the astroid given by $f(t) = [\cos^3(t), \sin^3(t)]$ for $t \in [0, 2\pi]$. Let $P(t)$ be a point on C. Let P_x and P_y be the x- and y-intercepts of the line tangent to C at $P(t)$. Show that the line segment $\overline{P_x P_y}$ has constant length; i.e., the length of the segment is independent of t. (*Click on the image to see a larger graph.*)



2. Let $\vec{r} : \mathbb{R} \to \mathbb{R}^3$ be a vector-valued function that has 2 continuous derivatives for all t. Prove or disprove

$$\frac{d}{dt}[\vec{r}(t) \times \vec{r}\,'(t)] = \vec{r}(t) \times \vec{r}\,''(t).$$

3. Let $f(t) = \dfrac{2t^2}{1+t^2}$ and set $C_{6\pi}$ to be the curve given by $[f(t)\,\cos(t), f(t)\,\sin(t)]$ for $t \in [0, 6\pi]$. Find the length of the curve C. Can you make a conjecture concerning the ratio $\dfrac{\text{length}(C_{2n\pi})}{4n}$ as $n \to \infty$?

4. Prove or disprove:
Let $A_1 = B^\circ([1,0], 1)$ and $A_{-1} = B^\circ([-1,0], 1)$ be open balls in $\mathbb{R}^2$. Then $E = A_1 \cup A_{-1}$ is not separated.

5. Let $f(x,y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$. Determine f_x and f_y. Is f differentiable at $(0,0)$?

6. A *harmonic function* is one that satisfies *Laplace's equation* $\nabla^2 f(x,y) = 0$ where $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$.

   (a) Prove that the functions

      i. $f(x,y) = x^3 - 3x\,y^2$

      ii. $g(x,y) = 3x^2\,y - y^3$

   are harmonic.

   (b) Find $\dfrac{d^2 z}{dt^2}$ for $z = x^3 - 3x\,y^2$ when $x(t) = \ln(t)$ and $y(t) = e^t$ without expanding z in terms of t.