ANALYSIS	Final Exam	NAME:________________
SUMMER '06		STUDENT ID #:________________

Work quickly and carefully, following directions closely. Answer all questions completely.

§I. PROBLEMS.

1. (a) Prove: If $a_k > 0$ $(\forall k \geq 0)$ and $\sum_k a_k$ converges, then $\sum_k 1/a_k$ diverges.

   (b) Is the condition that $a_k > 0$ necessary?

2. Suppose that $f : [a, b] \to \mathbb{R}$ is continous at all $x \in [a, b]$. Prove:
   If $\displaystyle\int_a^b |f(x)|\, dx = 0$, then $f(x) = 0$ on $[a, b]$.

3. Determine whether each of the sequences of functions given below converges pointwise, converges uniformly, or diverges. Justify your answer.

   (a) $f_n(x) = \dfrac{\tan^n(x)}{n}$ on $(-\pi/2, \pi/2)$.

   (b) $g_n(x) = \dfrac{x^n}{n!}$ on $\mathbb{R}$.

   (c) $h_n(x) = \dfrac{\sin(nx)}{\sqrt{n}}$ on $\mathbb{R}$.

   (d) $p_n(x) = \dfrac{1}{x^2 + n}$ on $[0, 1]$.

4. Archimedes discovered that the area under a parabolic arch is always $2/3 \times$ the base $\times$ the height.

   (a) Prove that Archimedes was correct. (You can assumer the parabola has form $y = -x^2 + ax$.)

   (b) Illustrate the result with the parabola $y = 8x - x^2$ on the interval $[0, 8]$.

5. Determine whether or not the following series converge. Justify your answers.

(a) $\displaystyle\sum_k \frac{1}{k\ \ln(k)}$

(b) $\displaystyle\sum_k \frac{(k!)^2}{(2k)!}$

(c) $\displaystyle\sum_k \left(1+\frac{1}{k}\right)^k$

(d) $\displaystyle\sum_k \frac{3^{2k+1}}{k^{2k}}$