Rules of Differentiation Summary

Definition

The function $f(x)$ has a derivative $f'(x)$ if and only if

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Results

Assume that u is a differentiable function of x. Then

$$(u^n)' = n\, u^{n-1} \cdot (u)' \tag{1}$$

$$(e^u)' = e^u \cdot (u)' \tag{2}$$

$$\Big(\ln(u)\Big)' = \frac{1}{u} \cdot (u)' \tag{3}$$

Reductions

Assume that u and v are differentiable functions of x and that c is a real number. Then

$$(c \cdot u)' = c \cdot (u)' \qquad \textit{Constant Multiple Rule} \tag{4}$$

$$(u \pm v)' = (u)' \pm (v)' \qquad \textit{Sum Rule} \tag{5}$$

$$(u \cdot v)' = (u)' \cdot v + u \cdot (v)' \qquad \textit{Product Rule} \tag{6}$$

$$\left(\frac{u}{v}\right)' = \frac{(u)' \cdot v - u \cdot (v)'}{v^2} \qquad \textit{Quotient Rule} \tag{7}$$

The Chain Rule I: Suppose that u and v are differentiable functions for all x. Then $u\big(v(x)\big)$ is differentiable, i.e., $(u \circ v)'$ exists, and

$$\frac{du}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx} \tag{8}$$

The Chain Rule II: Suppose that $f(x)$ and $g(x)$ are differentiable functions for all x. Then $f \circ g$ is a differentiable function and

$$f'\big(g(x)\big) = f'\big(g(x)\big) \cdot g'(x)$$

Problems

1. The Quotient Rule is a combination of the Product and Chain Rules. Fill in reasons each step is valid (from algebra or a derivative rule):

$$\left(\frac{u}{v}\right)' = \big(u \cdot (v)^{-1}\big)' \qquad \_\_\_\_\_\_\_\_\_\_\_\_$$

$$= (u)' \cdot v^{-1} + u \cdot \big(v^{-1}\big)' \qquad \_\_\_\_\_\_\_\_\_\_\_\_$$

$$= (u)' \cdot v^{-1} + u \cdot \big(-1 \cdot v^{-2} \cdot v'\big) \qquad \_\_\_\_\_\_\_\_\_\_\_\_$$

$$= \frac{u'}{v} - \frac{u \cdot v'}{v^2} \qquad \_\_\_\_\_\_\_\_\_\_\_\_$$

$$\left(\frac{u}{v}\right)' = \frac{(u)' \cdot v - u \cdot (v)'}{v^2} \qquad \_\_\_\_\_\_\_\_\_\_\_\_$$