OCSA Report

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Fall, 2016

Spring ’16 OCSA

1. Intro to Complex Numbers Monograph

   — *Some Projects can Crash & Burn*

   Software problems.

   - HTML 5 Canvas: Possible, but difficult; iBooks requires reinventing the wheel for each interactive diagram / display. $\left(\textit{filesize} \underset{t\to 0}{\longrightarrow} \infty\right)$
   - Javascript: Security holes, users deactivate, and proliferating libraries!
   - WebSketchPad: Works quite well in an iBook!
     “Unfortunately, we are not in a position to sell WSP to customers for commercial purposes.” Bryan Corwin, Product Manager.

Spring ’16 OCSA

2. The Good News
   - Partial Fractions: Developed a derivative technique and connections to Taylor/Laurent expansions.
     – *Paper submitted, revised per referees’ comments, resubmitted.*
   - Analytic Hierarchy Process (AHP) applied to 2 meter antenna selection.
     – *Paper submitted including spreadsheet implementing AHP.*
   - Leibniz’ Harmonic Triangle.
     – *Paper including student projects is in press.*
   - Function Design Recipes.
     – *In progress with Erica S.-Y.*
   - Complex Roots of Polynomials via Geometric Properties.
     – *With M. Bossé and H. Otey (undergrad), presented at TIME-2016, Mexico City. Three papers submitted, two more in progress.*

Main Aspects of the Projects, I

- Partial Fractions

$$A = \lim_{x\to a}(x-a)\cdot\left[\frac{A}{x-a}+\Phi(x)\right] = \lim_{x\to a}(x-a)\cdot\frac{p(x)}{q(x)} = \lim_{x\to a}\frac{p(x)}{\frac{q(x)-q(a)}{x-a}} = \frac{p(a)}{q'(a)}$$

  where a is real or complex.
  Similar equalities give the coefficients for higher multiplicity roots.

- AHP
  – Spreadsheet with complete implementation of AHP

- Leibniz’ Harmonic Triangle
  – Leibniz & Pascal and Leibniz’ original diagram

- Function Design Recipes
  – Function design recipe template

Main Aspects of the Projects, II

- Complex Roots of Polynomials via Geometric Properties — with Mike Bossé and Hunter Otey.
  - Propositions

Prop 1: Choose a base pt $x_0 \neq 0$. For the reduced quartic q,

$$q'(0) = \frac{q(x_0) - q(-x_0)}{2x_0}$$
$$q''(0) = \frac{q(x_0) - 2q(0) + q(-x_0)}{x_0^2} - 2x_0^2$$

Prop 2: Choose a base pt $x_0 \neq 0$. Given $q(x_0)$, $q(-x_0)$, and $q(0)$, the quartic's roots are symmetric pairs given by the *(large)* formula . . .

- Applets: WSP: Quintic and Higher Powers; Maple Circles and Lines

Papers Submitted and In-Progress

W. C. Bauldry.
Partial fractions via calculus.
Primus, (revised and resubmit), 2016.

W. C. Bauldry.
Choosing an antenna mathematically.
CQ Communications, (submitted), 2016.

W. C. Bauldry.
Dr. Leibniz and his amazing triangle.
Centroid, (to appear), 2016.

W. C. Bauldry, M. J. Bossé, and S. H. Otey.
A three point construction for the roots of a quartic polynomial.
College Math Journal, (submitted), 2016.

W. C. Bauldry, M. J. Bossé, and S. H. Otey.
Visualizing complex roots.
The Math Teacher, (revised and resubmit), 2016.

W. C. Bauldry, M. J. Bossé, and S. H. Otey.
Circle constructions of polynomial complex roots.
Electronic Journal of Mathematics and Technology, (submitted), 2016.

W. C. Bauldry and E. Slate-Young.
Function design recipes.
(*in progress*).

W. C. Bauldry, M. J. Bossé, and S. H. Otey.
Quintic polynomials and complex roots.
(in progress).

It Continues. . .

Definition

Set

$$Q_3 = \{q \in \mathbb{P}_5 \mid \text{monic with real roots } r_i,\, i = 1..3,\ \&\ \text{complex roots } z_\pm = a \pm b\imath\},$$

$$s_1 = r_1 + r_2 + r_3, \quad s_2 = r_1r_2 + r_1r_3 + r_2r_3, \quad s_3 = r_1r_2r_3$$

and

$$q_2 = q(2x_0),\ q_1 = q(x_0),\ q_0 = q(0),\ q_{-1} = q(-x_0),\ q_{-2} = q(-2x_0)$$

Theorem ("Five Point Proposition")

Let $q \in Q_3$. Choose a base point $x_0 \neq 0$. With s_i and q_j as defined above,

$$a = -\frac{1}{2}\left(s_1 + \frac{q_2 - 4q_1 + 6q_0 - 4q_{-1} + q_{-2}}{4!\, x_0^4}\right).$$

The complex roots lie on the intersections of the line $x = a$ and the circle of radius $R = \sqrt{-q_0/s_3}$ centered at the origin.

Update 9/13/16

General Theorem

As of Sept, 13, 2016, we have extended the theorem from quintic to all polynomials of degree greater than 3.

THEOREM. For $Q(J, x) = \left(\prod_{i=1}^{J}(x - r_i)\right)\left((x-a)^2 + b^2\right)$ with $J \geq 3$ real roots (counting multiplicities) and base point $x_0 \neq 0$ for $Q_\alpha = Q(\alpha x_0)$, we have

$$a = \frac{-1}{2}\left(\frac{1}{n!\, x_0^n} \cdot \sum_{k=0}^{n}(-1)^k \binom{n}{k} Q_{\lfloor n/2 \rfloor - k} + \sum_{i=1}^{J} r_i\right) - \cos^2\left(\frac{J\pi}{2}\right) \cdot \frac{n+2}{4} \cdot x_0$$

where $n = J + 1$ and

$$R^2 = (-1)^J Q(0)\left[\prod_{i=1}^{J} r_i\right]^{-1}$$

Whereupon the complex conjugate roots of Q lie on the intersections of the line $x = a$ and the circle with radius R centered at the origin.