# **OCSA** Report

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# Spring '16 OCSA

1. Intro to Complex Numbers Monograph

— Some Projects can Crash & Burn

Software problems.

- HTML 5 Canvas: Possible, but difficult; iBooks requires reinventing the wheel for each interactive diagram / display.  $\left(filesize \rightarrow \infty \right)$
- Javascript: Security holes, users deactivate, and proliferating libraries!
- WebSketchPad: Works quite well in an iBook! *"Unfortunately, we are not in a position to sell WSP to customers for commercial purposes."* Bryan Corwin, Product Manager.

# Spring '16 OCSA

#### 2. The Good News

- Partial Fractions: Developed a derivative technique and connections to Taylor/Laurent expansions.
   *Paper submitted, revised per referees' comments, resubmitted.*
- Analytic Hierarchy Process (AHP) applied to 2 meter antenna selection. – *Paper submitted including spreadsheet implementing AHP*.
- Leibniz' Harmonic Triangle. *Paper including student projects is in press.*
- Function Design Recipes. – In progress with Erica S.-Y.
- Complex Roots of Polynomials via Geometric Properties.
  With M. Bossé and H. Otey (undergrad), presented at TIME-2016, Mexico City. Three papers submitted, two more in progress.

OCSA Report: 2-7

# Main Aspects of the Projects, I

• Partial Fractions

$$A = \lim_{x \to a} (x - a) \cdot \left[ \frac{A}{x - a} + \Phi(x) \right] = \lim_{x \to a} (x - a) \cdot \frac{p(x)}{q(x)} = \lim_{x \to a} \frac{p(x)}{\frac{q(x) - q(a)}{x - a}} = \frac{p(a)}{q'(a)}$$

where *a* is real or complex.

Similar equalities give the coefficients for higher multiplicity roots.

• AHP

- Spreadsheet with complete implementation of AHP

- Leibniz' Harmonic Triangle
  Leibniz & Pascal and Leibniz' original diagram
- Function Design Recipes
  - Function design recipe template

### Main Aspects of the Projects, II

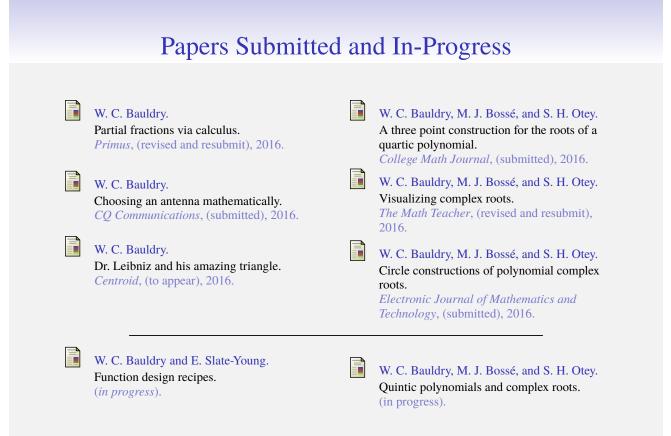
- Complex Roots of Polynomials via Geometric Properties

   with Mike Bossé and Hunter Otey.
  - Propositions
  - **Prop 1:** Choose a base pt  $x_0 \neq 0$ . For the reduced quartic q,

$$q'(0) = \frac{q(x_0) - q(-x_0)}{2x_0}$$
$$q''(0) = \frac{q(x_0) - 2q(0) + q(-x_0)}{x_0^2} - 2x_0^2$$

- Prop 2: Choose a base pt  $x_0 \neq 0$ . Given  $q(x_0)$ ,  $q(-x_0)$ , and q(0), the quartic's roots are symmetric pairs given by the (*large*) formula . . .
- Applets: WSP: Quintic and Higher Powers; Maple Circles and Lines

OCSA Report: 4-7



### It Continues...

#### Definition

Set

 $Q_3 = \{q \in \mathbb{P}_5 \mid \text{monic with real roots } r_i, i = 1..3, \& \text{ complex roots } z_{\pm} = a \pm b i \},\$ 

$$s_1 = r_1 + r_2 + r_3$$
,  $s_2 = r_1 r_2 + r_1 r_3 + r_2 r_3$ ,  $s_3 = r_1 r_2 r_3$ 

and

$$q_2 = q(2x_0), q_1 = q(x_0), q_0 = q(0), q_{-1} = q(-x_0), q_{-2} = q(-2x_0)$$

Theorem ("Five Point Proposition")

Let  $q \in Q_3$ . Choose a base point  $x_0 \neq 0$ . With  $s_i$  and  $q_j$  as defined above,

$$a = -\frac{1}{2} \left( s_1 + \frac{q_2 - 4q_1 + 6q_0 - 4q_{-1} + q_{-2}}{4! \, x_0^4} \right)$$

The complex roots lie on the intersections of the line x = a and the circle of radius  $R = \sqrt{-q_0/s_3}$  centered at the origin.

OCSA Report: 6-7

### Update 9/13/16

#### General Theorem

As of Sept, 13, 2016, we have extended the theorem from quintic to all polynomials of degree greater than 3.

THEOREM. For  $Q(J, x) = \left(\prod_{i=1}^{J} (x - r_i)\right) ((x - a)^2 + b^2)$  with  $J \ge 3$  real roots (counting multiplicities) and base point  $x_0 \ne 0$  for  $Q_\alpha = Q(\alpha x_0)$ , we have

$$a = \frac{-1}{2} \left( \frac{1}{n! \, x_0^n} \cdot \sum_{k=0}^n (-1)^k \binom{n}{k} Q_{\lfloor n/2 \rfloor - k} + \sum_{i=1}^J r_i \right) - \cos^2 \left( \frac{J\pi}{2} \right) \cdot \frac{n+2}{4} \cdot x_0$$

where n = J + 1 and

$$R^{2} = (-1)^{J} Q(0) \left[\prod_{i=1}^{J} r_{i}\right]^{-1}$$

Whereupon the complex conjugate roots of Q lie on the intersections of the line x = a and the circle with radius R centered at the origin.