

Connections Project

The table below summarizes the geometric connections between f , f' , and f'' .

$f(x)$		$f'(x)$		$f''(x)$
increasing	→	nonnegative		
max/min	→	root		
decreasing	→	nonpositive		
minimum	→	root	→	nonnegative
maximum	→	root	→	nonpositive
concave up	→	increasing	→	positive
inflection pt	→	max/min	→	root
concave down	→	decreasing	→	negative

Table 1: Y^e Charte

Problems

1. Give an example illustrating each row in the table. Show your function, its derivative(s), and appropriately labeled graph(s). (You may use one function's graphs to illustrate a group of rows of the chart.)
2. Where else, other than at a root of the derivative, can extrema occur? (Give sample graph(s).)
3. Suppose that $x = 1$ is a root of the derivative; i.e., $g'(1) = 0$. Does the original function $g(x)$ have to have an *extreme value* (maximum or minimum) at $x = 1$?
4. Suppose that $x = 2$ is a root of the second derivative; i.e., $h''(2) = 0$. Does the original function $h(x)$ have to have an *inflection point* at $x = 2$?

DEFINITION: A zero or root of f at $x = a$ has *multiplicity* n or *order* n if $f(a) = 0$, $f'(a) = 0$, $f''(a) = 0$, up to $f^{(n-1)}(a) = 0$, but $f^{(n)}(a) \neq 0$.

5. Show that $p(x) = x^4 - 7x^3 + 18x^2 - 20x + 8$ has a root of order 3 at $x = 2$ and of order 1 at $x = 1$.
6. Replace *root* in the second row of the chart with
 - (a) *odd root*. Does this change the implication?
 - (b) *even root*. Does this change the implication?

Odd and *even* denote a root of f of odd or even order n .
7. List your project team members: