Trigonometric Identities

Right-Triangle Definitions

$$\sin(\theta) = \text{Opp}/\text{Hyp}$$
$$\tan(\theta) = \text{Opp}/\text{Adj}$$
$$\sec(\theta) = \text{Hyp}/\text{Adj}$$

$$\cos(\theta) = \text{Adj}/\text{Hyp}$$
$$\cot(\theta) = \text{Adj}/\text{Opp}$$
$$\csc(\theta) = \text{Hyp}/\text{Opp}$$

Radians $\iff$ Degrees

$$x_r = \frac{\pi_r}{180^\circ} \cdot t^\circ$$

$$t^\circ = \frac{180^\circ}{\pi_r} \cdot x_r$$

Reduction Formulas

$$\sin(-x) = -\sin(x)$$
$$\sin\left(\frac{\pi}{2} - x\right) = \cos(x)$$
$$\sin\left(\frac{\pi}{2} + x\right) = \cos(x)$$
$$\sin(\pi - x) = \sin(x)$$
$$\sin(\pi + x) = -\sin(x)$$

$$\cos(-x) = \cos(x)$$
$$\cos\left(\frac{\pi}{2} - x\right) = \sin(x)$$
$$\cos\left(\frac{\pi}{2} + x\right) = -\sin(x)$$
$$\cos(\pi - x) = -\cos(x)$$
$$\cos(\pi + x) = -\cos(x)$$

Pythagorean Theorem

$$\sin^2(x) + \cos^2(x) = 1 \qquad \tan^2(x) + 1 = \sec^2(x) \qquad 1 + \cot^2(x) = \csc^2(x)$$

Sum and Difference Formulas

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$$
$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\beta)\sin(\alpha)$$
$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \sin(\beta)\cos(\alpha)$$
$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\beta)\sin(\alpha)$$
$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

Double Angle and Half Angle Formulas

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$
$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$
$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

$$\sin(\theta/2) = \pm\sqrt{\frac{1 - \cos(\theta)}{2}}$$
$$\cos(\theta/2) = \pm\sqrt{\frac{1 + \cos(\theta)}{2}}$$
$$\tan(\theta/2) = \frac{1 - \cos(\theta)}{\sin(\theta)} = \frac{\sin(\theta)}{1 - \cos(\theta)}$$

Euler's Identity

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

Derivative Formulas

$$\frac{d}{dx}\sin(x) = \cos(x) \qquad \frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx}\sinh(x) = \cosh(x) \qquad \frac{d}{dx}\text{arcsinh}(x) = \frac{1}{\sqrt{1 + x^2}}$$

$$\frac{d}{dx}\tan(x) = \sec^2(x) \qquad \frac{d}{dx}\arctan(x) = \frac{1}{1 + x^2} \qquad \frac{d}{dx}\tanh(x) = \text{sech}^2(x) \qquad \frac{d}{dx}\text{arctanh}(x) = \frac{1}{1 - x^2}$$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x) \qquad \frac{d}{dx}\text{arcsec}(x) = \frac{1}{x\sqrt{x^2 - 1}} \qquad \frac{d}{dx}\text{sech}(x) = \text{sech}(x)\tanh(x) \qquad \frac{d}{dx}\text{arcsech}(x) = \frac{1}{x\sqrt{1 - x^2}}$$