

Analysis for Teachers with Maple

Wm C Bauldry

Appalachian State University

ICTCM 19
February, 2007

1 Introduction

2 Course Design: Problem Based

1 Calculus

- 1 Brief calculus review
- 2 Calculus problems

2 Analysis

- 1 Basic Problems
- 2 Supplementary Problems
- 3 Enrichment Problems

3 History of Analysis

4 Student presentations & reports

3 Maple in the Course

1 Calculus

- 1 Graphs
- 2 Limits
- 3 Derivatives
- 4 Integrals
- 5 Sequences & Series

2 Analysis

- 1 Series
- 2 Counter examples
- 3 limits, continuity, & uniform continuity via graphs
- 4 Sequence and series of functions
- 5 Lebesgue measure
- 6 Special functions

4 Questions/Comments

- Objective of the course
 - Enhance mathematics teacher content knowledge
 - Many undergraduate programs do not require analysis/advanced calculus
 - More students take calculus in high school, than in post-secondary schools
- Place in the curriculum
 - Required course in Masters in Math., Secondary Ed., or MAT programs
 - Prerequisite: BS in Math Ed or Math plus teaching experience
- Offering
 - Four or five week summer session. (5 days per week; 2 hours, 40 minutes per day.)

Course Design: Problem Based

1 Syllabus (four week)

Week 1 Review of Calculus, Outline of a Calculus Course, Links to AP Calculus

Week 2 Analysis Problems (3 to 4 per day)

Week 3 Analysis Problems

Week 4 Readings, History, Special Topics (proofs, topology, &c), Presentations, Final Report

2 Individual Project: Write a paper on a historical analysis topic; make a presentation to the class

3 Class Projects:

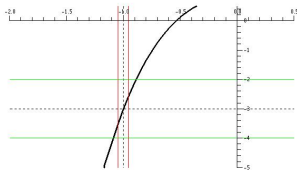
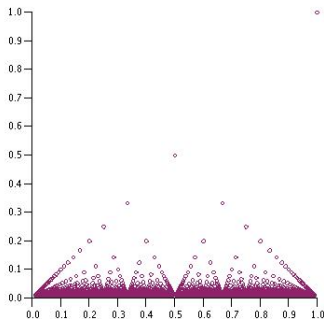
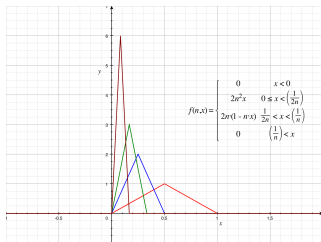
- Make an “Analysis Concept Map”
- Build a glossary of analysis terms
- Develop an annotated analysis bibliography

4 Assessment

Maple in the Course: Calculus

Calculus

- 1 Graphs
- 2 Limits with ϵ - δ
- 3 Derivatives with difference quotients
- 4 Integrals via definition
- 5 Sequences & Series



Maple in the Course: Tutors

Precalculus – Function Slope Tutor

File Help

Plot Window

Enter a function and point of tangency

$f(x) = x^2 - 1$ $x = 1$

Slopes of Secant Lines

Point	Slope
-4.00	-3.00
-1.50	-1.500
-.250	.750
.375	1.38
.688	1.69
6.00	7.00
3.50	4.50
2.25	3.25
1.62	2.62
1.31	2.31

Tangent Line

Point of contact:
[1, 0]

Slope of tangent line:
2.

Equation of tangent line:
 $y = 2*x - 2$.

Speed: + - Play Display Animate Plot Options Close

Maple Command
FunctionSlopePlot(x^2-1, 1, 'view'=[-4.50..6.50, -5.22..25], animation);

Multivariate Calculus – Taylor Approximation

File Help

Plot Window

Enter a function and initial point

$f(x,y) = \sqrt{x^2 + y^2 + 2}$

[x , y] = [0 , 0]

Order: 3

Taylor Polynomial

The Taylor polynomial for $1/2*x^2 + 2*y^2 + 4*(1/2)$ of order 3 about the point $(x, y) = (0, 0)$ is:

$$1/2*x^2 + 1/4*y^2 + 2 + 1$$

Display Animate Plot Options Close

Maple Command
TaylorApproximation(1/2*x^2+2*y^2+4*(1/2), [x, y] = [0, 0], 3, view = [-4..4, -4..4, 0..10], output = plot);

Precalculus – Rational Functions

File Help

Rational Function $r(x) = p(x)/q(x)$

$p(x) = 3*x^2 + 2*x - 1$
 $q(x) = x^3 - x + 2$

Equations of Asymptotes

Vertical asymptote(s) located at
 $x = [-9, -1521379707]$

Horizontal asymptote located at $y = 0$

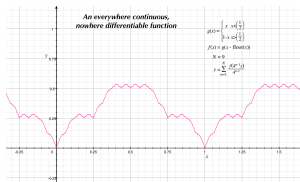
Display Plot Options Close

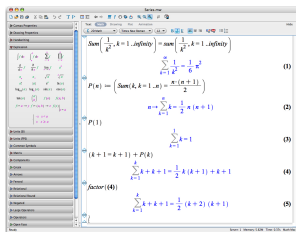
Maple Command
RationalFunctionPlot((3*x^2+2*x-1)/(x^3-x+2), view = [-5..5, -5..5]);

Maple in the Course: Analysis

Analysis

- 1 Sequences and series
- 2 Counter examples
- 3 Continuity and uniform continuity
- 4 Sequence and series of functions
- 5 Special functions





$$\text{Sum}\left(\frac{1}{k^2}, k=1..n, \text{info}\right) = \text{ans}\left(\frac{1}{k^2}, k=1..n, \text{info}\right)$$
$$\sum_{k=1}^n \frac{1}{k^2} = \frac{1}{6} n^3 - \frac{1}{2} n^2$$
 (1)

$$P(n) := \left(\text{Sum}(k, k=1..n)\right) = \frac{n(n+1)}{2}$$

$$n = \sum_{k=1}^n k = \frac{1}{2} n(n+1)$$
 (2)

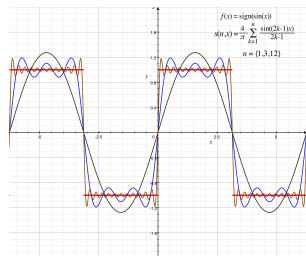
$$\sum_{k=1}^n k = 1$$
 (3)

$$(k+1) = k+1 + P(k)$$

$$\sum_{k=1}^n k+k+1 = \frac{1}{2} k(k+1) + k+1$$
 (4)

factor(4)

$$\sum_{k=1}^n k+k+1 = \frac{1}{2} (k+2)(k+1)$$
 (5)



Maple in the Course: Ratio & Root Tests

The screenshot shows the Maple software interface with a document titled "untitled". The interface includes a menu bar with options like Text, Math, Drawing, Plot, and Animation. A toolbar below the menu bar contains various icons for editing and calculation. On the left side, there is a sidebar with a list of tool palettes: Canvas Properties, Drawing Properties, Handwriting, Expression, Units (SI), Units (FPS), Common Symbols, Matrix, Components, Greek, Arrows, Fenced, Relational, Relational Round, Negated, Large Operators, Operators, Open Face, and Fraktur. The main workspace contains several mathematical expressions and limits, numbered (1) through (5) on the right side. The expressions are:

- (1) $r(n) := \text{abs}\left(\frac{a(n+1)}{a(n)}\right)$
$$n \rightarrow \left| \frac{a(n+1)}{a(n)} \right|$$
- (2) $a(n) := \frac{n^n}{n!} :$
 $\text{Sum}(a(n), n = 1 .. \text{infinity}) = \text{sum}(a(n), n = 1 .. \text{infinity})$
$$\sum_{n=1}^{\infty} \frac{n^n}{n!} = \sum_{n=1}^{\infty} \frac{n^n}{n!}$$
- (3) $\text{Limit}(r(n), n = \text{infinity}) = \text{limit}(r(n), n = \text{infinity})$
$$\lim_{n \rightarrow \infty} \left| \frac{(n+1)^{n+1} n!}{(n+1)! n^n} \right| = e$$
- (4) $\rho(n) := a(n)^{\left(\frac{1}{n}\right)}$
$$n \rightarrow a(n)^{\frac{1}{n}}$$
- (5) $\text{Limit}(\rho(n), n = \text{infinity}) = \text{limit}(\rho(n), n = \text{infinity})$
$$\lim_{n \rightarrow \infty} \left(\frac{n^n}{n!} \right)^{\frac{1}{n}} = e$$

At the bottom of the window, the status bar shows "Ready" on the left and "Server: 1 Memory: 116.41M Time: 26.21s Math Moc" on the right.

Maple in the Course: Series and Induction

The screenshot shows the Maple software interface with a document titled "Series.mw". The left sidebar contains various toolbars for Canvas Properties, Drawing Properties, Handwriting, and Expression. The main workspace displays a series of mathematical equations and commands:

- Equation (1):
$$\text{Sum}\left(\frac{1}{k^2}, k = 1 .. \text{infinity}\right) = \text{sum}\left(\frac{1}{k^2}, k = 1 .. \text{infinity}\right)$$
$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{1}{6} \pi^2 \quad (1)$$
- Equation (2):
$$P(n) := \left(\text{Sum}(k, k = 1 .. n) = \frac{n \cdot (n + 1)}{2}\right)$$
$$n \rightarrow \sum_{k=1}^n k = \frac{1}{2} n (n + 1) \quad (2)$$
- Equation (3):
$$P(1)$$
$$\sum_{k=1}^1 k = 1 \quad (3)$$
- Equation (4):
$$(k + 1 = k + 1) + P(k)$$
$$\sum_{k=1}^k k + k + 1 = \frac{1}{2} k (k + 1) + k + 1 \quad (4)$$
- Equation (5):
$$\text{factor}((4))$$
$$\sum_{k=1}^k k + k + 1 = \frac{1}{2} (k + 2) (k + 1) \quad (5)$$

The status bar at the bottom indicates "Ready" and provides system information: "Server: 1 Memory: 5.62M Time: 0.37s Math Moc".

Questions? Comments?

Thank you.

Materials available at

Slides: <http://www.mathsci.appstate.edu/~wmcbl/ICTCM19/>

A course link:

<http://www.mathsci.appstate.edu/~wmcbl/Class/archive.html>