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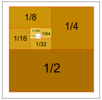
$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} ... = \lim_{n\to\infty} \sum_{i=0}^{n-1} \frac{1}{2}\frac{1}{2}^i = \frac{.5}{1-.5}$ 

$\sum_{i=0}^{\infty} \frac{1}{2}\frac{1}{2}^i = \sum_{i=1}^{\infty} \frac{1}{2}^i$

Zeno's Paradox

Drug Doses, Periodic Payments and More

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Geometric Series? Constant ratio between consecutive terms?

yes—common ratio is .04 and starting value is 250

after n^{th} dose: $\displaystyle\sum_{i=0}^{n-1} 250(.04)^i = \frac{a(1-x^n)}{1-x} = \frac{250(1-.04^n)}{1-.04}$

$|x| = .04 < 1$ so $\displaystyle\sum_{i=0}^{\infty} 250 \cdot .04^i$ converges to $\dfrac{250}{1-.04} \approx 260.42$

Clicker Question

1. Is this geometric? $5 - 10 + 20 - 40 + 80...$

a) yes and I have a good reason why
b) yes but I am unsure of why
c) no, although it is a series
d) no, it is a sequence, not a series

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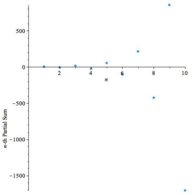

$\sum_{i=0}^{\infty} 5(-2)^i$ $\quad |x| = 2 > 1$ diverges

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2. Is this geometric? $2(.1)^2 + 2(.1)^3 + ... + 2(.1)^{11}$

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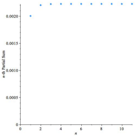
finite series always converge! $\sum_{i=0}^{9} (2(.1)^2).1^i$



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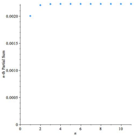

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$|.1|$, the common ratio, < 1, so geo series converges to

$$\frac{a(1-x^n)}{1-x} = (2(.1)^2)\frac{1-.1^{10}}{1-.1}$$

Clicker Question

3. We deposit \$150 per month (at the end of each month) into an account that pays 1.2% each month. What do we have in 3 years if the interest rate doesn't change?

a) $\sum_{i=0}^{35} 150(1+.012)^i$

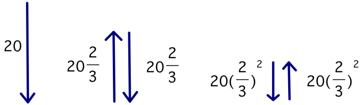
b) $\dfrac{150(1-1.012^{36})}{1-1.012}$

c) both of the above

d) none of the above

Clicker Question

4. We drop a ball from 20 ft and the ball bounces 2/3 as high each time as the last. Can the total vertical distance (up and down) after the n^{th} bounce hits the ground be written as a geometric series?



a) yes

b) no

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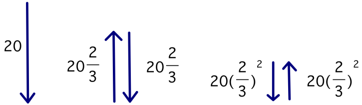
4. We drop a ball from 20 ft and the ball bounces 2/3 as high each time as the last. Can the total vertical distance (up and down) after the n^{th} bounce hits the ground be written as a geometric series?



a) yes

b) no

$$\sum_{i=0}^{n} 40\frac{2}{3}^{i} -20 = \frac{40(1-\frac{2}{3}^{n+1})}{1-\frac{2}{3}} - 20$$

History and Applications

- Archimedes: compute the area enclosed by a parabola and a straight line using an infinite number of triangles and sum of geometric series
- early calculus: series represented geometric quantities and were manipulated using methods extended from finite procedures
- geometric series arise in many places, like in the examples we mentioned
- physical chemistry such as harmonic oscillator
- important to the study of Taylor series, via comparison