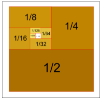
# 9.2 Geometric Series

- series: add the terms in a sequence
- geometric series—ratio between any two consecutive terms is constant: $\sum_{i=0}^{n-1} ax^i = a + ax + ax^2 + ... + ax^{n-1}$
- sum of the first $n$ terms? $\frac{a(1-x^n)}{1-x}$. Careful of # terms and starting index
- finite geo series always converges.
- $\infty$ geo series converge? $\lim_{n\to\infty} \frac{a(1-x^n)}{1-x}$ if $|x| < 1$ : $\frac{a}{1-x}$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} ... = \lim_{n\to\infty} \sum_{i=0}^{n-1} \frac{1}{2}\frac{1}{2}^i = \frac{.5}{1-.5}$ 

$\sum_{i=0}^{\infty} \frac{1}{2}\frac{1}{2}^i = \sum_{i=1}^{\infty} \frac{1}{2}^i$

Zeno's Paradox

# *Drug Doses, Periodic Payments and More*

250 mg every 6 hours, when 4% of the drug remains. How much is in the body after the $n^{th}$ dose? Does the infinite series converge (i.e. stabilize in the body)?

Geometric Series? Constant ratio between consecutive terms?

## *Discussion Question*

1. Is this geometric? $5 - 10 + 20 - 40 + 80...$

a) yes and I have a good reason why
b) yes but I am unsure of why
c) no, although it is a series
d) no, it is a sequence, not a series

# *Discussion Question*

2. Is this geometric? $2(.1)^2 + 2(.1)^3 + ... + 2(.1)^{11}$

a) yes and I have a good reason why
b) yes but I am unsure of why
c) no, although it is a series
d) no, it is a sequence, not a series

# *Discussion Question*

3. We deposit \$150 per month (at the end of each month) into an account that pays 1.2% each month. What do we have in 3 years if the interest rate doesn't change?

a) $\sum_{i=0}^{35} 150(1 + .012)^i$

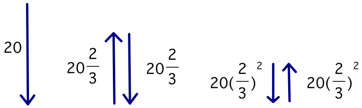
b) $\dfrac{150(1 - 1.012^{36})}{1 - 1.012}$

c) both of the above

d) none of the above

# *Discussion Question*

4. We drop a ball from 20 ft and the ball bounces 2/3 as high each time as the last. Can the total vertical distance (up and down) after the $n^{th}$ bounce hits the ground be expressed as almost a geometric series?




- a) yes
- b) no

# *History and Applications*

- Archimedes: compute the area enclosed by a parabola and a straight line using an infinite number of triangles and sum of geometric series
- early calculus: series represented geometric quantities and were manipulated using methods extended from finite procedures
- geometric series arise in many places, like in the examples we mentioned
- physical chemistry such as harmonic oscillator
- important to the study of Taylor series, via comparison