9.2 Geometric Series versus 9.3 p-Series

- ratio between any two consecutive terms is constant.

  sum of the first n terms: $\dfrac{a(1-x^n)}{1-x}$. Careful of # terms and starting index. $\lim_{n\to\infty} \dfrac{a(1-x^n)}{1-x} = \dfrac{a}{1-x}$ if $|x|<1$

- $\sum_{n=1}^{\infty} \dfrac{1}{n^p}$ converges if $p>1$ and diverges if $p \leq 1$.

$\sum_{n=1}^{\infty} (\frac{1}{2})^n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \ldots$ geo series, $|x| = .5 < 1$ conv to $\dfrac{.5}{1-.5}$

$\sum_{n=1}^{\infty} \dfrac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} \ldots$ p series: $p = 2 > 1$ conv by integral test: terms dec +:

$\int_1^\infty \frac{1}{x^2}\,dx = \lim_{b\to\infty} \int_1^b x^{-2}\,dx = \lim_{b\to\infty} \frac{x^{-1}}{-1}\Big|_1^b = 0 - -1$

$1 \leq \sum_{n=1}^{\infty} \frac{1}{n^2} \leq 1+$ first term $= 1 + \frac{1}{1^2} = 1+1$

Sequence versus Series

1. Is this a geometric series? yes no

   Geometric Series: $\sum_{i=0}^{\infty} ax^i$ where x is the common ratio and a is a constant. $\sum_{i=0}^{n} ax^i = \frac{a(1-x^{n+1})}{1-x}$.

   $\sum_{i=0}^{\infty} ax^i = \frac{a}{1-x}$ provided $|x| < 1$.

2. Can we apply the Terms not Going to 0? yes no

   Terms not Going to 0: For $\sum a_n$, if the $\lim_{n->\infty} a_n \neq 0$, then the infinite series does not converge.

3. Are the terms decreasing and positive eventually, and if so is this an integral we can do? yes no

   Integral Test: For $\sum a_n$, if the terms are decreasing and $a_n > 0$, then the series behaves the same way as $\int_a^{\infty} a_n dn$, & $\int_a^{\infty} f(x)dx \leq \sum a_n \leq$ 1st term $+ \int_a^{\infty} f(x)dx$.