

7.4 Partial Fractions (Quotients of Polynomials)

- Useful when denominator divides up into real factors that are linear or irreducible quadratic (or repeated)
- Based on adding fractions via a common denominator

What I want you to show me... the expansion, and the system of linear equations to solve for A, B, C...

The diagram illustrates the process of partial fraction decomposition. On the right, the rational function $\frac{5x-4}{x^2-x-2}$ is shown. A blue arrow points from this function to a question mark, and another blue arrow points from the question mark to the sum of two partial fractions: $\frac{2}{x-2} + \frac{3}{x+1}$. The two partial fractions are each enclosed in a light blue circle. Below the partial fractions, the text "Partial Fractions" is written in blue, with two blue arrows pointing upwards to the denominators $x-2$ and $x+1$.

Factor	Term in Partial Fraction Decomposition
$(ax + b)$	$\frac{A}{ax+b}$

Clicker Question

1. Which of the following integrals can be integrated using partial fractions via subdividing into a product of linear terms like x or $x + \sqrt{2}$ or similar? (Hint: keep factoring, if possible)

a) $\int \frac{1}{x^4 - 3x^2 + 2} dx$

b) $\int \frac{1}{x^3 - 4x} dx$

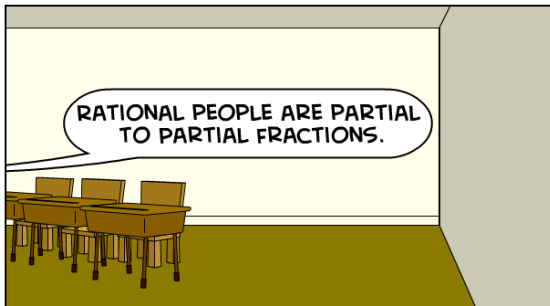
c) $\int \frac{1}{1 + x^2} dx$

d) all of the above

e) exactly two of a), b), c)

PARTIAL FRACTION INTEGRATION

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**technically,
the glass is always
full.**

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$$\frac{2}{x-2} + \frac{3}{x+1} \quad \leftarrow ? \quad \frac{5x-4}{x^2-x-2}$$

Partial Fractions

Factor
($ax + b$)

Term in Partial Fraction Decomposition
 $\frac{A}{ax+b}$

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Factor
 $(ax + b)$
 $(ax + b)^2$

Term in Partial Fraction Decomposition

$$\frac{A}{ax+b} + \frac{B}{(ax+b)^2}$$

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Partial Fractions

Factor

$$(ax + b)$$

$$(ax + b)^2$$

$$(ax^2 + bx + c)$$

Term in Partial Fraction Decomposition

$$\frac{A}{ax+b} + \frac{B}{(ax+b)^2}$$
$$\frac{Ax+B}{ax^2+bx+c}$$



Clicker Question

2. If a denominator is irreducible quadratic, then what does the numerator in the partial fraction decomposition look like?

- a) $Ax + B$
- b) A
- c) $Ax^2 + Bx + C$
- d) No way to tell, depends on the problem
- e) None of the above

Clicker Question

3. What is a useful method for $\int \frac{1}{1-x^2} dx$?

- a) Integration by w-substitution
- b) Integration by parts
- c) Integration by partial fractions
- d) More than one of the above

History and Applications

- Early 1700s Johann Bernoulli investigated $\frac{a^2}{a^2-x^2}$ which he solved by partial fractions. Gottfried Wilhelm Leibniz discovered them independently
- British physicist Oliver Heaviside (1850-1925) introduced partial fraction expansions as part of his “operational calculus”
- Inverse Laplace Transform, which in turn is extremely useful in places like electronics and signal processing, physics, astroengineering
- Analyzing linear differential systems such as resonant circuits and feedback-control systems
- Law of Mass Action relates time a chemical takes to create a product (in molecules) from two ingredients