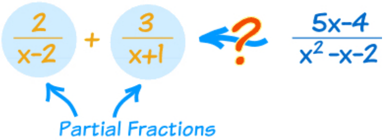
7.4 Partial Fractions (Quotients of Polynomials)

- Useful when denominator divides up into real factors that are linear or irreducible quadratic (or repeated)
- Based on adding fractions via a common denominator

What I want you to show me... the expansion, and the system of linear equations to solve for A, B, C...



Factor	Term in Partial Fraction Decomposition
$(ax+b)$	$\frac{A}{ax+b}$

Clicker Question

1. Which of the following integrals can be integrated using partial fractions via subdividing into a product of linear terms like x or $x + \sqrt{2}$ or similar? (Hint: keep factoring, if possible)

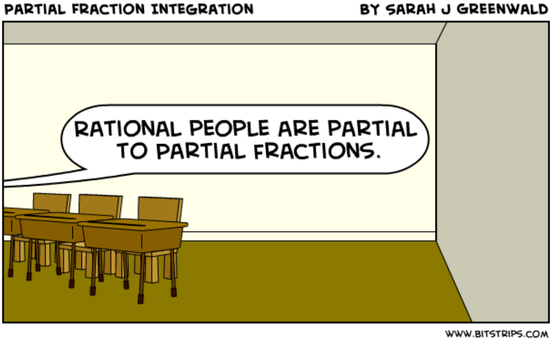
a) $\displaystyle\int \frac{1}{x^4 - 3x^2 + 2}\,dx$

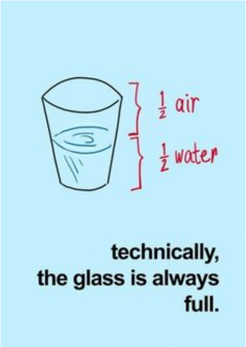
b) $\displaystyle\int \frac{1}{x^3 - 4x}\,dx$

c) $\displaystyle\int \frac{1}{1 + x^2}\,dx$

d) all of the above

e) exactly two of a), b), c)


PARTIAL FRACTION INTEGRATION
BY SARAH J GREENWALD
RATIONAL PEOPLE ARE PARTIAL TO PARTIAL FRACTIONS.
WWW.BITSTRIPS.COM


1/2 air
1/2 water
technically, the glass is always full.

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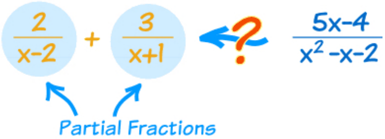

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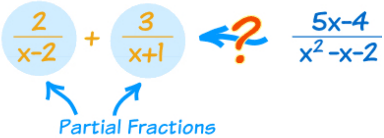

Factor	Term in Partial Fraction Decomposition
$(ax+b)$	$\frac{A}{ax+b}$
$(ax+b)^2$	$\frac{A}{ax+b} + \frac{B}{(ax+b)^2}$

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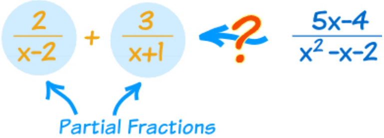

Factor	Term in Partial Fraction Decomposition
$(ax+b)$	$\frac{A}{ax+b}$
$(ax+b)^2$	$\frac{A}{ax+b}+\frac{B}{(ax+b)^2}$
(ax^2+bx+c)	$\frac{Ax+B}{ax^2+bx+c}$

Clicker Question

2. If a denominator is irreducible quadratic, then what does the numerator in the partial fraction decomposition look like?

a) $Ax + B$

b) A

c) $Ax^2 + Bx + C$

d) No way to tell, depends on the problem

e) None of the above

Clicker Question

3. What is a useful method for $\int \frac{1}{1-x^2}\,dx$?

a) Integration by w-substitution

b) Integration by parts

c) Integration by partial fractions

d) More than one of the above

History and Applications

- Early 1700s Johann Bernoulli investigated $\frac{a^2}{a^2-x^2}$ which he solved by partial fractions. Gottfried Wilhelm Leibniz discovered them independently
- British physicist Oliver Heaviside (1850-1925) introduced partial fraction expansions as part of his "operational calculus"
- Inverse Laplace Transform, which in turn is extremely useful in places like electronics and signal processing, physics, astroengineering
- Analyzing linear differential systems such as resonant circuits and feedback-control systems
- Law of Mass Action relates time a chemical takes to create a product (in molecules) from two ingredients