9.5: Power Series: Sums of Powers of x or $(x-a)$

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  Ex3: ~~$(x-0)^1 + (x-1)^2 + (x-2)^3 + ... + (x-n)^{n+1} + ...$~~

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- Still series so apply 9.2, 9.3 & 9.4
- Ratio test or geometric series test is often helpful
- Convergence may depend on constraining x: r radius of convergence, interval of convergence $(a-r, a+r)$ might include 1 or both endpoints so check with 9.2–9.4 methods

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Clicker Question

1. Can we use the geometric series test to check the radius of convergence of $\sum_{n=0}^{\infty}(5x)^n = 1 + 5x + 25x^2 + 125x^3 + \ldots$?

a) yes

b) no we must use the ratio test instead

Infinitely many people walk into a bar one at a time. The first person orders $\frac{1}{2}$ beer, the second orders $\frac{1}{4}$, the n^{th} orders $\frac{1}{2^n}$.
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pours $\frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$ beer and tells them to share.

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2. Find the radius of convergence for $\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + ...$

a) the power series converges for no x (i.e. always diverges)
b) the power series converges for only $x = 0$, so $r = 0$
c) the power series converges for all x so $r = \infty$
d) $r = 1$
e) other

Do not underestimate the power of the power series.

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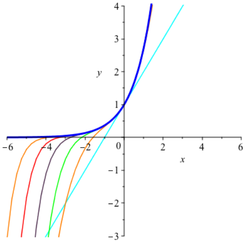


$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \ldots$$

Clicker Question

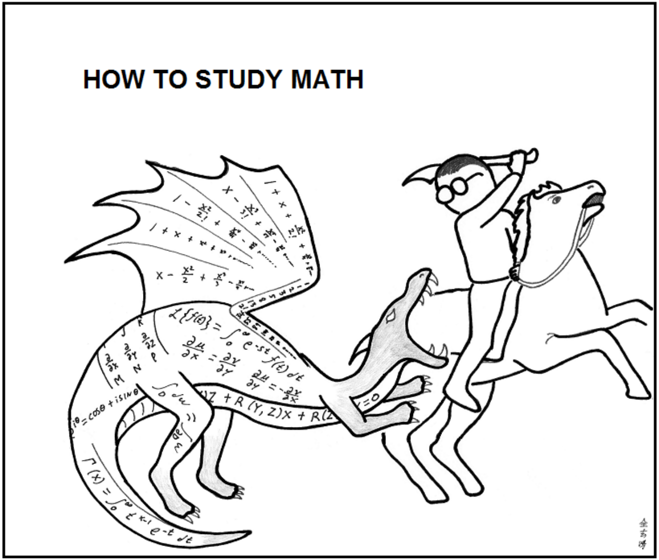
3. The ratio test can be used to show that $r = 1$ for $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$, which has a center of 0. Determine the interval of convergence by checking the endpoints ($x = -1$ and $x = 1$).

a) the power series converges for $-1 < x < 1$, i.e. $(-1, 1)$
b) the power series converges for $-1 \leq x \leq 1$, i.e. $[-1, 1]$
c) the power series converges for $-1 \leq x < 1$, i.e. $[-1, 1)$
d) the power series converges for $-1 < x \leq 1$, i.e. $(-1, 1]$
e) none of the above

Clicker Question

4. Suppose the power series $\sum_{n=0}^{\infty} c_n x^n$ converges if $x = -3$ and diverges if $x = 7$. Which of the following are true?

a) The power series converges if $x = 0$
b) The power series converges if $x = 3$
c) The power series diverges if $x = 3$
d) a) and b)
e) a) and c)



Don't just read it; fight it!

--- Paul R. Halmos

Image: `https://abstrusegoose.com/353`

History and Applications

- James Gregory (1671) For $-1 \leq x \leq 1$,
$$\arctan(x) = \sum_{n=1}^{\infty} (-1)^{(n-1)} \frac{x^{2n-1}}{2n-1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$$
- analysis, number theory and combinatorics
- electrical engineering (Z-transform)
- Planetary motion and the shape of a vibrating drumhead: Bessel functions, named for Wilhelm Bessel (1817). The Bessel function of order 0 is defined by

$$\sum_{n=0}^{\infty} \frac{-1^n x^{2n}}{2^{2n}(n!)^2}$$

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