# 7.1 Integration by Substitution
# Group Work Target Practice

Evaluate each of the following integrals in groups of two or three.

One can be done using substitution—show me: $w$, $dw$, $\int$ with respect to $w$ [Hint: Try to find $w$ so that $dw$ is in $\int$. It is often helpful to choose $w$ "inside" of some other function]

For one of the integrals it is not possible to find an antiderivative using any method, so identify which one this is.

For one of the integrals substitution won't work, but an algebraic method from calculus 1 will.

1. $\displaystyle\int_0^2 e^{x^2}\,dx.$

   $w$-subs won't work because if $w = x^2$, then $dw = 2xdx$, but there is no $x$ to take advantage of. None of our methods work here. In fact, this is not an elementary integral—no nice closed form exists (so when these come up in real life, people use approximation methods like 7.5 and chapter 9 and 10).

2. $\displaystyle\int \frac{2x-3}{x^2}dx$

   The is a known integral from calculus 1. We can divide the numerator up (careful this wouldn't work for a denominator!) and then use the fact that those Riemann sums are additive, so the integral of a sum is the sum of integrals, as follows:

   $$=\int \frac{2x}{x^2} + \frac{-3}{x^2}dx = \int \frac{2x}{x^2}dx + \int \frac{-3}{x^2}dx$$
   $$= \int \frac{2}{x}dx + \int -3x^{-2}dx = 2\ln|x| - 3\frac{x^{-1}}{-1} + C = 2\ln|x| - -3x^{-1} + C = 2\ln|x| + \frac{3}{x} + C$$

3. $\displaystyle\int \frac{e^{\frac{1}{x}}}{x^2}dx$

   This is $w$-subs because we have a function inside another function, and its derivative is also there to make use of:

   Let $w = \frac{1}{x} = x^{-1}$. Then $dw = -x^{-2}dx = -\frac{1}{x^2}dx$. We are only missing the negative sign, so put that (constant) on the other side: $-dw = \frac{1}{x^2}dx$.

   $$\int e^w \times -dw = -\int e^w dw = -e^w + C = -e^{\frac{1}{x}} + C$$