7.2 Integration by Parts
Group Work Target Practice

Evaluate each of the following integrals in groups of two or three. For one of the integrals it is a straight parts. For one of the integrals you must use substitution after you use parts.

1. $\int \frac{\ln x}{x^5}\,dx$

   detail: algebraic x^{-5} is before log $\ln x$, so the derivative (d) from detail shows $v' = x^{-5}$.

   $u = \ln x \qquad v' = x^{-5}$

   $u' = \frac{1}{x} \qquad v = \frac{x^{-4}}{-4}$

   $uv - \int u'v dx = \ln x \frac{x^{-4}}{-4} - \int \frac{1}{x}\frac{x^{-4}}{-4}dx = \ln x \frac{x^{-4}}{-4} - \int \frac{1}{-4}\frac{x^{-4}}{x}dx$

   (by the rules of exponents) $= \ln x \frac{x^{-4}}{-4} - \int \frac{x^{-4-1}}{-4}dx$

   $= \frac{\ln x}{-4x^4} - \int \frac{x^{-5}}{-4}dx = \frac{\ln x}{-4x^4} - \int \frac{1}{-4}x^{-5}dx = \frac{\ln x}{-4x^4} - \frac{1}{-4}\frac{x^{-4}}{(-4)} + c = \frac{\ln x}{-4x^4} - \frac{1}{16x^4} + c$

2. $\int \arcsin x\,dx$

   There are only a few applications of parts like this where $v' = 1$: $\int \ln x dx$ from class and inverse trig functions like this one.

   detail: algebraic 1 is before inverse trig $\arcsin x$, so the derivative (d) from detail shows $v' = 1$.

   $u = \arcsin x \qquad v' = 1$

   $u' = \frac{1}{\sqrt{1-x^2}} \qquad v = x$

   $uv - \int u'v dx = x \arcsin x - \int x\frac{1}{\sqrt{1-x^2}}dx$

   This integral can be completed using $w - sub$, with $w = 1 - x^2$, and $dw = -2xdx$ since we have the x to work with. The 2 goes on the other side (we have a negative from parts we can use): $\frac{dw}{2} = -xdx$

   $= x \arcsin x - \int x\frac{1}{\sqrt{1-x^2}}dx = x \arcsin x + \int \frac{1}{\sqrt{w}}\frac{dw}{2} = x \arcsin x + \int w^{-\frac{1}{2}}\frac{dw}{2}$

   $= x \arcsin x + \frac{w^{\frac{1}{2}}}{\frac{1}{2}}\frac{1}{2} = x \arcsin x + w^{\frac{1}{2}} = x \arcsin x + \sqrt{1-x^2} + c$