7.4 Partial Fractions Group Work Target Practice

Work in groups of two or three.

1. For $\dfrac{2}{s^4-1}$ write out the factors and the generic forms for the numerators, but do not solve for the constants.

   $s^4 - 1 = (s^2-1)(s^2+1) = (s-1)(s+1)(s^2+1)$ is broken up into linear and irreducible quadratic pieces. In partial fractions, each linear factor (including any repeated linear terms) gets a constant as the numerator, and each irreducible quadratic gets a linear numerator. In this case we have:
   $\dfrac{2}{s^4-1} = \dfrac{A}{s-1} + \dfrac{B}{s+1} + \dfrac{Cs+D}{s^2+1}$
   While I didn't ask you to progress further for this problem to find the linear equations, nor solve for the constants, I wanted to comment on the generic methods of integration that could arise here. Integrals of the form $\frac{A}{s-1}$ and $\frac{B}{s+1}$ are each $w-$subs. To integrate the last term we would generally break it up into a sum of integrals. However in this specific example C will actually be 0 (if it weren't then its integral would be another $w-$subs) The D integral is a calc 1 integral for $\arctan(s)$

2. Solve for $\displaystyle\int \frac{3x+11}{x^2-x-6}\,dx$ using the method of partial fractions. Show work.

   $x^2 - x - 6 = (x-3)(x+2)$ is broken up into linear and irreducible quadratic pieces In partial fractions, each linear factor (including any repeated linear terms) gets a constant as the numerator, and each irreducible quadratic gets a linear numerator. In this case we have:
   $\dfrac{3x+11}{(x-3)(x+2)} = \dfrac{A}{x-3} + \dfrac{B}{x+2}$
   To solve for A and B multiply through by the denominator of the left side: (x-3)(x+2).

   $3x + 11 = \dfrac{A}{x-3}(x-3)(x+2) + \dfrac{B}{x+2}(x-3)(x+2) = A(x+2) + B(x-3)$
   Next multiply out and then collect like terms from the left and ride hand sides of the equations, to create linear equations in terms of the coefficients A and B:

   $3x + 11 = Ax + 2A + Bx - 3B$

   x terms: $3 = A + B$

   constant terms: $11 = 2A - 3B$

   Solve this linear system for the constants. For calc 2, substitution works fine. If you go on to take linear algebra, elimination methods will be the focus there.

   From eq 1: $A = 3 - B$, sub in to eq 2: $11 = 2(3-B) - 3B = 6 - 5B$, so $B = -1$. Sub back into eq 1: $A = 4$.
   $\displaystyle\int \frac{3x+11}{x^2-x-6}dx = \int \frac{4}{x-3}dx + \int \frac{-1}{x+2}dx = 4\ln|x-3| - \ln|x+2| + c$, where the last integrals work because of $w = x \pm a$, where a is constant, then $dw = dx$, so in each case we have numbers times $\int \frac{dw}{w}$.